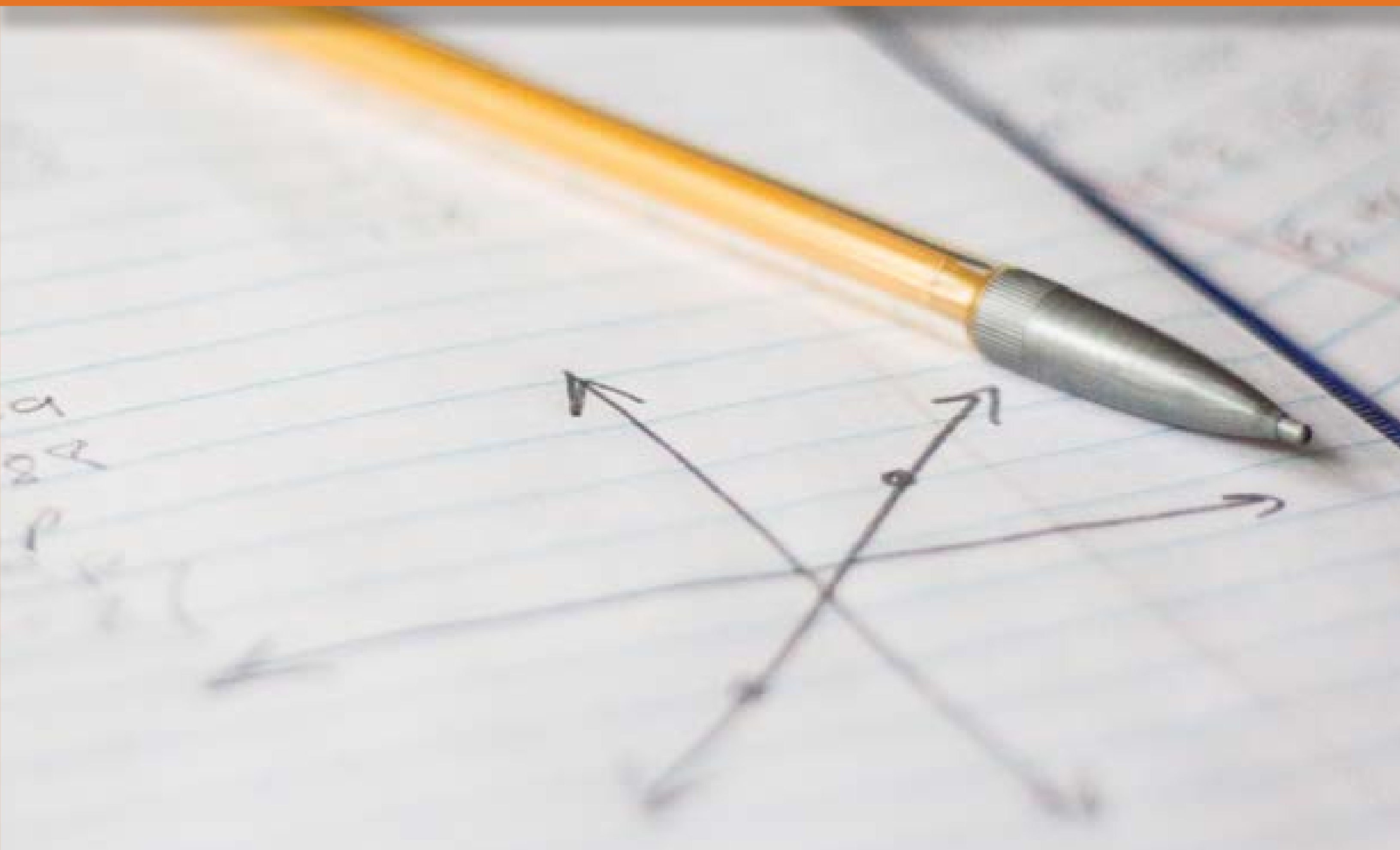
CollegeBoard Pre-AP®

Pre-AP® Algebra 1

COURSE GUIDE

INCLUDES

- Approach to teaching and learning
- Course map
- Course framework
- Sample assessment questions



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COURSE GUIDE

Updated Fall 2020

Please visit Pre-AP online at **preap.org** for more information and updates about the course and program features.

ABOUT COLLEGE BOARD

College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, College Board was created to expand access to higher education. Today, the membership association is made up of over 6,000 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success—including the SAT® and the Advanced Placement Program®. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools.

For further information, visit **www.collegeboard.org**.

PRE-AP EQUITY AND ACCESS POLICY

College Board believes that all students deserve engaging, relevant, and challenging grade-level coursework. Access to this type of coursework increases opportunities for all students, including groups that have been traditionally underrepresented in AP and college classrooms. Therefore, the Pre-AP program is dedicated to collaborating with educators across the country to ensure all students have the supports to succeed in appropriately challenging classroom experiences that allow students to learn and grow. It is only through a sustained commitment to equitable preparation, access, and support that true excellence can be achieved for all students, and the Pre-AP course designation requires this commitment.

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About Pre-AP

Introduction to Pre-AP

Every student deserves classroom opportunities to learn, grow, and succeed. College Board developed Pre-AP® to deliver on this simple premise. Pre-AP courses are designed to support all students across varying levels of readiness. They are not honors or advanced courses.

Participation in Pre-AP courses allows students to slow down and focus on the most essential and relevant concepts and skills. Students have frequent opportunities to engage deeply with texts, sources, and data as well as compelling higher-order questions and problems. Across Pre-AP courses, students experience shared instructional practices and routines that help them develop and strengthen the important critical thinking skills they will need to employ in high school, college, and life. Students and teachers can see progress and opportunities for growth through varied classroom assessments that provide clear and meaningful feedback at key checkpoints throughout each course.

DEVELOPING THE PRE-AP COURSES

Pre-AP courses are carefully developed in partnership with experienced educators, including middle school, high school, and college faculty. Pre-AP educator committees work closely with College Board to ensure that the course resources define, illustrate, and measure grade-level-appropriate learning in a clear, accessible, and engaging way. College Board also gathers feedback from a variety of stakeholders, including Pre-AP partner schools from across the nation who have participated in multiyear pilots of select courses. Data and feedback from partner schools, educator committees, and advisory panels are carefully considered to ensure that Pre-AP courses provide all students with grade-level-appropriate learning experiences that place them on a path to college and career readiness.

PRE-AP PROGRAM COMMITMENTS

The Pre-AP Program asks participating schools to make four commitments:

1. **Pre-AP for All:** Pre-AP frameworks and assessments serve as the foundation for all sections of the course at the school.
2. **Course Frameworks:** Teachers align their classroom instruction with the Pre-AP course frameworks.
    - Schools commit to provide the core resources to ensure Pre-AP teachers and students have the materials they need to engage in the course.

3. **Assessments:** Teachers administer at least one learning checkpoint per unit on Pre-AP Classroom and four performance tasks.
4. **Professional Learning:** Teachers complete the foundational professional learning (Online Foundational Modules or Pre-AP Summer Institute) and at least one online performance task scoring module. The current Pre-AP coordinator completes the Pre-AP Coordinator Online Module.

PRE-AP EDUCATOR NETWORK

Similar to the way in which teachers of Advanced Placement® (AP®) courses can become more deeply involved in the program by becoming AP Readers or workshop consultants, Pre-AP teachers also have opportunities to become active in their educator network. Each year, College Board expands and strengthens the Pre-AP National Faculty—the team of educators who facilitate Pre-AP Professional Learning Workshops. Pre-AP teachers can also become curriculum and assessment contributors by working with College Board to design, review, or pilot the course resources.

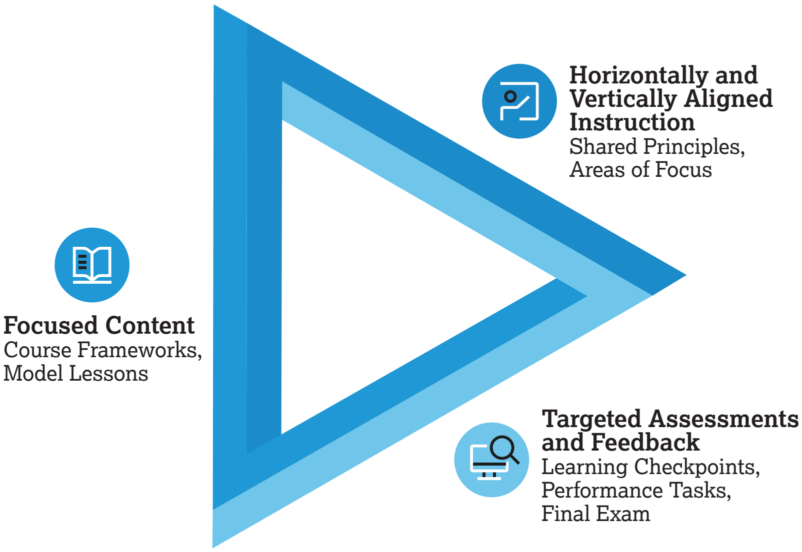
HOW TO GET INVOLVED

Schools and districts interested in learning more about participating in Pre-AP should visit **preap.org/join** or contact us at **preap@collegeboard.org**.

Teachers interested in becoming members of Pre-AP National Faculty or participating in content development should visit **preap.org/national-faculty** or contact us at **preap@collegeboard.org**.

Pre-AP Approach to Teaching and Learning

Pre-AP courses invite all students to learn, grow, and succeed through focused content, horizontally and vertically aligned instruction, and targeted assessments for learning. The Pre-AP approach to teaching and learning, as described below, is not overly complex, yet the combined strength results in powerful and lasting benefits for both teachers and students. This is our theory of action.



FOCUSED CONTENT

Pre-AP courses focus deeply on a limited number of concepts and skills with the broadest relevance for high school coursework and college and career success. The course framework serves as the foundation of the course and defines these prioritized concepts and skills. Pre-AP model lessons and assessments are based directly on this focused framework. The course design provides students and teachers with intentional permission to slow down and focus.

HORIZONTALLY AND VERTICALLY ALIGNED INSTRUCTION

Shared principles cut across all Pre-AP courses and disciplines. Each course is also aligned to discipline-specific areas of focus that prioritize the critical reasoning skills and practices central to that discipline.

SHARED PRINCIPLES

All Pre-AP courses share the following set of research-supported instructional principles. Classrooms that regularly focus on these cross-disciplinary principles allow students to effectively extend their content knowledge while strengthening their critical thinking skills. When students are enrolled in multiple Pre-AP courses, the horizontal alignment of the shared principles provides students and teachers across disciplines with a shared language for their learning and investigation, and multiple opportunities to practice and grow. The critical reasoning and problem-solving tools students develop through these shared principles are highly valued in college coursework and in the workplace.

Close Observation and Analysis

Students are provided time to carefully observe one data set, text, image, performance piece, or problem before being asked to explain, analyze, or evaluate. This creates a safe entry point to simply express what they notice and what they wonder. It also encourages students to slow down and capture relevant details with intentionality to support more meaningful analysis, rather than rush to completion at the expense of understanding.

Higher-Order Questioning

Students engage with questions designed to encourage thinking that is elevated beyond simple memorization and recall. Higher-order questions require students to make predictions, synthesize, evaluate, and compare. As students grapple with these questions, they learn that being inquisitive promotes extended thinking and leads to deeper understanding.

Evidence-Based Writing

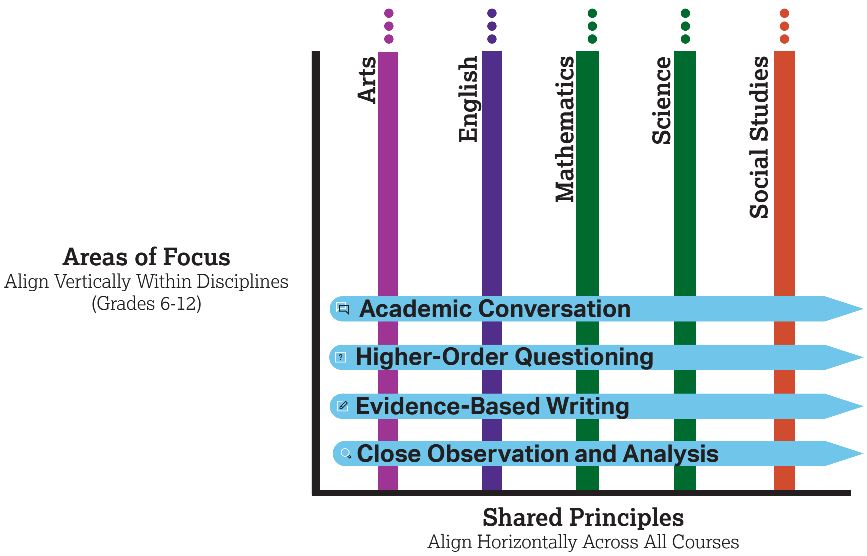
With strategic support, students frequently engage in writing coherent arguments from relevant and valid sources of evidence. Pre-AP courses embrace a purposeful and scaffolded approach to writing that begins with a focus on precise and effective sentences before progressing to longer forms of writing.

Academic Conversation

Through peer-to-peer dialogue, students' ideas are explored, challenged, and refined. As students engage in academic conversation, they come to see the value in being open to new ideas and modifying their own ideas based on new information. Students grow as they frequently practice this type of respectful dialogue and critique and learn to recognize that all voices, including their own, deserve to be heard.

AREAS OF FOCUS

The areas of focus are discipline-specific reasoning skills that students develop and leverage as they engage with content. Whereas the shared principles promote horizontal alignment across disciplines, the areas of focus provide vertical alignment within a discipline, giving students the opportunity to strengthen and deepen their work with these skills in subsequent courses in the same discipline.



For information about the Pre-AP mathematics areas of focus, see page 13.

TARGETED ASSESSMENTS FOR LEARNING

Pre-AP courses include strategically designed classroom assessments that serve as tools for understanding progress and identifying areas that need more support. The assessments provide frequent and meaningful feedback for both teachers and students across each unit of the course and for the course as a whole. For more information about assessments in Pre-AP Algebra 1, see page 53.

Pre-AP Professional Learning

As part of the program commitments, Pre-AP teachers agree to engage in two professional learning opportunities:

1. The first commitment is designed to help prepare teachers to teach their specific course. There are two options to meet this commitment: the Pre-AP Summer Institute (Pre-APSI) and the Online Foundational Modules. Both options provide continuing education units upon completion.
   - The Pre-AP Summer Institute provides a collaborative experience that empowers participants to prepare and plan for their Pre-AP course. While attending, teachers engage with Pre-AP course frameworks, shared principles, areas of focus, and sample model lessons. Participants are given supportive planning time where they work with peers to begin building their Pre-AP course plan.
   - Online Foundational Modules are available to all teachers of Pre-AP courses. In their 12- to 20-hour asynchronous course, teachers explore course materials and experience model lessons from the student's point of view. They also begin building their Pre-AP course plan.
2. The second professional learning opportunity helps teachers prepare for the performance tasks. As part of this commitment, teachers agree to complete at least one online performance task scoring module. Online scoring modules offer guidance and practice applying scoring guidelines and examining student work. Teachers may complete the modules independently or with teachers of the same course in their school's professional learning communities.

About Pre-AP Algebra 1

Introduction to Pre-AP Algebra 1

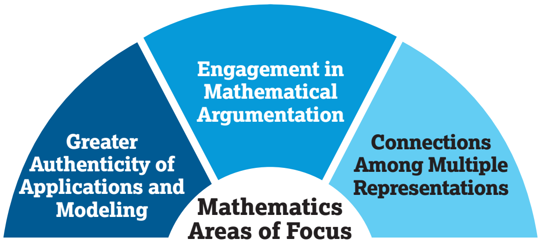
The Pre-AP Algebra 1 course is designed to deepen students' understanding of linear relationships by emphasizing patterns of change, multiple representations of functions and equations, modeling real world scenarios with functions, and methods for finding and representing solutions of equations and inequalities. Taken together, these ideas provide a powerful set of conceptual tools that students can use to make sense of their world through mathematics.

Rather than seeking to cover all topics traditionally included in a standard algebra textbook, this course focuses on the foundational algebraic knowledge and skills that matter most for college and career readiness. The Pre-AP Algebra 1 Course Framework highlights how to guide students to connect core ideas within and across the units of the course, promoting a coherent understanding of linear relationships.

The components of this course have been crafted to prepare not only the next generation of mathematicians, scientists, programmers, statisticians, and engineers, but also a broader base of mathematically informed citizens who are well equipped to respond to the array of mathematics-related issues that impact our lives at the personal, local, and global level.

PRE-AP MATHEMATICS AREAS OF FOCUS

The Pre-AP mathematics areas of focus, shown below, are mathematical practices that students develop and leverage as they engage with content. They were identified through educator feedback and research about where students and teachers need the most curriculum support. These areas of focus are vertically aligned to the mathematical practices embedded in other mathematics courses in high school, including AP, and in college, giving students multiple opportunities to strengthen and deepen their work with these skills throughout their educational career. They also support and align to the AP Calculus Mathematical Practices, the AP Statistics Course Skills, and the mathematical practices listed in various state standards.



Greater Authenticity of Applications and Modeling

Students create and use mathematical models to understand and explain authentic scenarios.

Mathematical modeling is a process that helps people explore, represent, analyze, and explain the world. In Pre-AP Algebra 1, students explore real-world contexts where mathematics can be used to make sense of a situation. They engage in the modeling process by making choices about what aspects of the situation to model, assessing how well the model represents the available data, drawing conclusions from their model, justifying decisions they make through the process, and identifying what the model helps clarify and what it does not.

In addition to mathematical modeling, Pre-AP Algebra 1 students engage in mathematics through authentic applications. Applications are similar to modeling problems in that they are drawn from real-world phenomena, but they differ because the applications dictate the appropriate mathematics to use to solve the problem. Pre-AP Algebra 1 balances these two types of real-world tasks.

Engagement in Mathematical Argumentation

Students use evidence to craft mathematical conjectures and prove or disprove them.

Conjecture and proof lie at the heart of the discipline of mathematics. Mathematics is both a way of thinking and a set of tools for solving problems. Pre-AP Algebra 1 students gain experience, comfort, and proficiency with mathematical thinking by observing and generalizing patterns in number sequences, graphs, equations, operations, and functions. They harness their curiosity to create problems to solve and conjectures to prove or disprove. Through mathematical argumentation, students learn how to be critical of their own reasoning and the reasoning of others.

Connections Among Multiple Representations

Students represent mathematical concepts in a variety of forms and move fluently among the forms.

Mathematical concepts can be represented in a variety of forms. Pre-AP Algebra 1 students learn how the multiple representations of a concept are connected to each other and how to fluently translate between graphical, numerical, algebraic, and verbal representations. Every mathematical representation illuminates certain characteristics of a concept while also obscuring other aspects. With experience that begins to develop in Pre-AP Algebra 1, students develop a nuanced understanding of which representations best serve a particular purpose.

PRE-AP ALGEBRA 1 AND CAREER READINESS

The Pre-AP Algebra 1 course resources are designed to expose students to a wide range of career opportunities that depend upon Algebra 1 knowledge and skills. Examples include not only field-specific careers such as mathematician or statistician but also other endeavors where algebraic knowledge is relevant and applicable, such as actuaries, engineers, programmers, carpenters, and HVAC technicians.

Career clusters that involve mathematics, along with examples of careers in mathematics and other careers that require the use of algebra, are provided below. Teachers should consider discussing these with students throughout the year to promote motivation and engagement.

Career Clusters Involving Mathematics
arts, A/V technology, and communications architecture and construction business management and administration finance government and public administration health science information technology manufacturing marketing STEM (science, technology, engineering, and math)

Examples of Mathematics Careers	**Examples of Algebra 1 Related Careers**
actuary financial analyst mathematician mathematics teacher professor programmer statistician	carpenter computer programmer economist electrician engineer HVAC technician operations research analyst programmer

Source for Career Clusters: "Advanced Placement and Career and Technical Education: Working Together." Advance CTE and the College Board. October 2018. **https://careertech.org/resource/ap-cte-working-together**.

For more information about careers that involve mathematics, teachers and students can visit and explore the College Board's Big Future resources: **https://bigfuture.collegeboard.org/majors/math-statistics-mathematics**.

SUMMARY OF RESOURCES AND SUPPORTS

Teachers are strongly encouraged to take advantage of the full set of resources and supports for Pre-AP Algebra 1, which is summarized below. Some of these resources are part of the Pre-AP Program commitments that lead to Pre-AP Course Designation. To learn more about the commitments for course designation, see details below and on page 68.

COURSE FRAMEWORK

Included in this guide as well as in the *Pre-AP Algebra 1 Teacher Resources*, the framework defines what students should know and be able to do by the end of the course. It serves as an anchor for model lessons and assessments, and it is the primary resource needed to plan the course. **Teachers commit to aligning their classroom instruction with the course framework**. *For more details see page 20.*

MODEL LESSONS

Teacher resources, available in print and online, include a robust set of model lessons that demonstrate how to translate the course framework, shared principles, and areas of focus into daily instruction. **Use of the model lessons is encouraged**. *For more details see page 51.*

LEARNING CHECKPOINTS

Accessed through Pre-AP Classroom (the Pre-AP digital platform), these short formative assessments provide insight into student progress. They are automatically scored and include multiple-choice and technology-enhanced items with rationales that explain correct and incorrect answers. **Teachers commit to administering one learning checkpoint per unit**. *For more details see page 53.*

PERFORMANCE TASKS

Available in the printed teacher resources as well as on Pre-AP Classroom, performance tasks allow students to demonstrate their learning through extended problem-solving, writing, analysis, and/or reasoning tasks. Scoring guidelines are provided to inform teacher scoring, with additional practice and feedback suggestions available in online modules on Pre-AP Classroom. **Teachers commit to using each unit's performance task**. *For more details see page 55.*

PRACTICE PERFORMANCE TASKS

Available in the student resources, with supporting materials in the teacher resources, these tasks provide an opportunity for students to practice applying skills and knowledge as they would in a performance task, but in a more scaffolded environment. **Use of the practice performance tasks is encouraged**. *For more details see page 55.*

FINAL EXAM

Accessed through Pre-AP Classroom, the final exam serves as a classroom-based, summative assessment designed to measure students' success in learning and applying the knowledge and skills articulated in the course framework. **Administration of the final exam is encouraged**. *For more details see page 62.*

PROFESSIONAL LEARNING

Both the Pre-AP Summer Institute (Pre-APSI) and the Online Foundational Modules support teachers in preparing and planning to teach their Pre-AP course. **All Pre-AP teachers make a commitment to either attend the Pre-APSI (in person or virtually) or complete the Online Foundational Modules. In addition, teachers agree to complete at least one Online Performance Task Scoring module**. *For more details see page 9.*

Course Map

PLAN

The course map shows how components are positioned throughout the course. As the map indicates, the course is designed to be taught over 140 class periods (based on 45-minute class periods), for a total of 28 weeks.

Model lessons are included for approximately 50% of the total instructional time, with the percentage varying by unit. Each unit is divided into key concepts.

TEACH

The model lessons demonstrate how the Pre-AP shared principles and mathematics areas of focus come to life in the classroom.

Shared Principles
Close observation and analysis
Higher-order questioning
Evidence-based writing
Academic conversation

Areas of Focus
Greater authenticity of applications and modeling
Engagement in mathematical argumentation
Connections among multiple representations

ASSESS AND REFLECT

Each unit includes two learning checkpoints and a performance task. These formative assessments are designed to provide meaningful feedback for both teachers and students.

Note: The final exam, offered during a six-week window in the spring, is not represented in the map.

UNIT 1 **Linear Functions and Linear Equations**

~45 Class Periods

Pre-AP model lessons provided for approximately 55% of instructional time in this unit

KEY CONCEPT 1.1
Constant Rate of Change and Slope

KEY CONCEPT 1.2
Linear Functions

Learning Checkpoint 1

KEY CONCEPT 1.3
Linear Equations

KEY CONCEPT 1.4
Linear Models of Nonlinear Scenarios

Learning Checkpoint 2

KEY CONCEPT 1.5
Two-Variable Linear Inequalities

Performance Task for Unit 1

UNIT 2 Systems of Linear Equations and Inequalities

~25 Class Periods

Pre-AP model lessons provided for approximately 30% of instructional time in this unit

KEY CONCEPT 2.1
The Solution to a System of Equations

KEY CONCEPT 2.2
Solving a System of Linear Equations Algebraically

KEY CONCEPT 2.3
Modeling with Systems of Linear Equations

Learning Checkpoint 1

KEY CONCEPT 2.4
Systems of Linear Inequalities

Learning Checkpoint 2

Performance Task for Unit 2

UNIT 3 Quadratic Functions

~45 Class Periods

Pre-AP model lessons provided for approximately 45% of instructional time in this unit

KEY CONCEPT 3.1
Functions with a Linear Rate of Change

KEY CONCEPT 3.2
The Algebra and Geometry of Quadratic Functions

Learning Checkpoint 1

KEY CONCEPT 3.3
Solving Quadratic Equations

KEY CONCEPT 3.4
Modeling with Quadratic Functions

Learning Checkpoint 2

Performance Task for Unit 3

UNIT 4 Exponent Properties and Exponential Functions

~25 Class Periods

Pre-AP model lessons provided for approximately 40% of instructional time in this unit

KEY CONCEPT 4.1
Exponent Rules and Properties

KEY CONCEPT 4.2
Roots of Real Numbers

Learning Checkpoint 1

KEY CONCEPT 4.3
Sequences with Multiplicative Patterns

KEY CONCEPT 4.4
Exponential Growth and Decay

Learning Checkpoint 2

Performance Task for Unit 4

Pre-AP Algebra 1 Course Framework

INTRODUCTION

Based on the Understanding by Design® (Wiggins and McTighe) model, the Pre-AP Algebra 1 Course Framework is back mapped from AP expectations and aligned to essential grade-level expectations. The course framework serves as a teacher's blueprint for the Pre-AP Algebra 1 instructional resources and assessments.

The course framework was designed to meet the following criteria:

- **Focused:** The framework provides a deep focus on a limited number of concepts and skills that have the broadest relevance for later high school, college, and career success.
- **Measurable:** The framework's learning objectives are observable and measurable statements about the knowledge and skills students should develop in the course.
- **Manageable:** The framework is manageable for a full year of instruction, fosters the ability to explore concepts in depth, and enables room for additional local or state standards to be addressed where appropriate.
- **Accessible:** The framework's learning objectives are designed to provide all students, across varying levels of readiness, with opportunities to learn, grow, and succeed.

COURSE FRAMEWORK COMPONENTS

The Pre-AP Algebra 1 Course Framework includes the following components:

Big Ideas

The big ideas are recurring themes that allow students to create meaningful connections between course concepts. Revisiting the big ideas throughout the course and applying them in a variety of contexts allows students to develop deeper conceptual understandings.

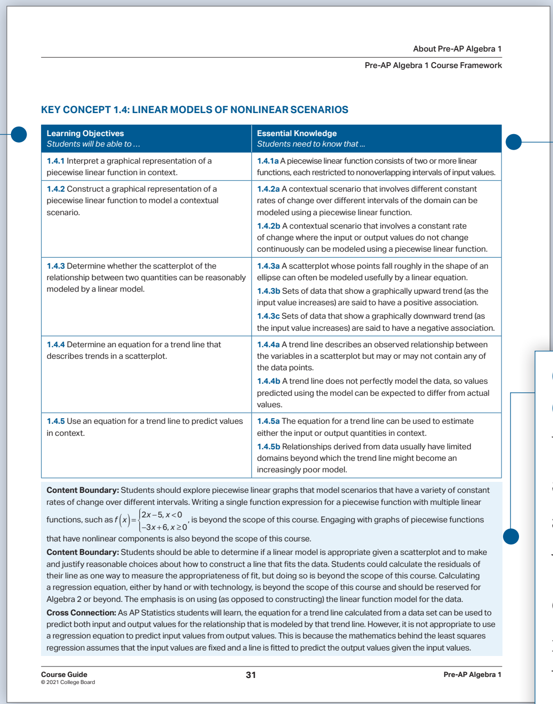
Enduring Understandings

Each unit focuses on a small set of enduring understandings. These are the long-term takeaways related to the big ideas that leave a lasting impression on students. Students build and earn these understandings over time by exploring and applying course content throughout the year.

Key Concepts

To support teacher planning and instruction, each unit is organized by key concepts. Each key concept includes relevant **learning objectives** and **essential knowledge statements** and may also include **content boundary and cross connection statements**. These are illustrated and defined below.

Learning Objectives:
These objectives define what a student needs to be able to do with essential knowledge to progress toward the enduring understandings. The learning objectives serve as actionable targets for instruction and assessment.



About Pre-AP Algebra 1

Pre-AP Algebra 1 Course Framework

KEY CONCEPT 1.4: LINEAR MODELS OF NONLINEAR SCENARIOS

Learning Objectives *Students will be able to …*	Essential Knowledge *Students need to know that …*
1.4.1 Interpret a graphical representation of a piecewise linear function in context.	**1.4.1a** A piecewise linear function consists of two or more linear functions, each restricted to nonoverlapping intervals of input values.
1.4.2 Construct a graphical representation of a piecewise linear function to model a contextual scenario.	**1.4.2a** A contextual scenario that involves different constant rates of change over different intervals of the domain can be modeled using a piecewise linear function. **1.4.2b** A contextual scenario that involves a constant rate of change where the input or output values do not change continuously can be modeled using a piecewise linear function.
1.4.3 Determine whether the scatterplot of the relationship between two quantities can be reasonably modeled by a linear model.	**1.4.3a** A scatterplot whose points fall roughly in the shape of an ellipse can often be modeled usefully by a linear equation. **1.4.3b** Sets of data that show a graphically upward trend (as the input value increases) are said to have a positive association. **1.4.3c** Sets of data that show a graphically downward trend (as the input value increases) are said to have a negative association.
1.4.4 Determine an equation for a trend line that describes trends in a scatterplot.	**1.4.4a** A trend line describes an observed relationship between the variables in a scatterplot but may or may not contain any of the data points. **1.4.4b** A trend line does not perfectly model the data, so values predicted using the model can be expected to differ from actual values.
1.4.5 Use an equation for a trend line to predict values in context.	**1.4.5a** The equation for a trend line can be used to estimate either the input or output quantities in context. **1.4.5b** Relationships derived from data usually have limited domains beyond which the trend line might become an increasingly poor model.

Content Boundary: Students should explore piecewise linear graphs that model scenarios that have a variety of constant rates of change over different intervals. Writing a single function expression for a piecewise function with multiple linear functions, such as $f(x)=\begin{cases}2x-5, x<0\\-3x+6, x\geq 0\end{cases}$, is beyond the scope of this course. Engaging with graphs of piecewise functions that have nonlinear components is also beyond the scope of this course.

Content Boundary: Students should be able to determine if a linear model is appropriate given a scatterplot and to make and justify reasonable choices about how to construct a line that fits the data. Students could calculate the residuals of their line as one way to measure the appropriateness of fit, but doing so is beyond the scope of this course. Calculating a regression equation, either by hand or with technology, is beyond the scope of this course and should be reserved for Algebra 2 or beyond. The emphasis is on using (as opposed to constructing) the linear function model for the data.

Cross Connection: As AP Statistics students will learn, the equation for a trend line calculated from a data set can be used to predict both input and output values for the relationship that is modeled by that trend line. However, it is not appropriate to use a regression equation to predict input values from output values. This is because the mathematics behind the least squares regression assumes that the input values are fixed and a line is fitted to predict the output values given the input values.

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Pre-AP Algebra 1

Essential Knowledge Statements:
Each essential knowledge statement is linked to a learning objective. One or more essential knowledge statements describe the knowledge required to perform each learning objective.

Content Boundary and Cross Connection Statements:
When needed, content boundary statements provide additional clarity about the content and skills that lie within versus outside of the scope of this course.

Cross connection statements highlight important connections that should be made between key concepts within and across the units.

BIG IDEAS IN PRE-AP ALGEBRA 1

While the Pre-AP Algebra 1 framework is organized into four core units of study, the content is grounded in four big ideas, which are cross-cutting concepts that build conceptual understanding and spiral throughout the course. Since these ideas cut across units, they serve as the underlying foundation for the enduring understandings, key concepts, and learning objectives that make up the focus of each unit. A deep and productive understanding of the concepts presented in Pre-AP Algebra 1 relies on these four big ideas:

- **Patterns of Change:** Families of functions are uniquely defined by their patterns of change. Linear functions have a constant rate of change, quadratic functions have a linear rate of change, and exponential functions have a constant multiplicative rate of change.
- **Representations:** Functions and equations can be represented graphically, numerically (in tables), algebraically (with symbols), or verbally (in words). Algebraic forms of functions and equations can be purposefully manipulated into equivalent forms to reveal certain aspects of the function/equation.
- **Modeling with Functions:** Functions can be used to model real-world phenomena. A function derived from a real-world context can be manipulated free of its context, but the solution must be translated back in order to interpret its meaning in context.
- **Solutions:** A solution to an equation or inequality is a value or set of values that makes the equation or inequality true. Solutions can be found by applying rules of algebra to symbolic expressions, examining a graph of the equation or inequality, or testing numerical values.

OVERVIEW OF PRE-AP ALGEBRA 1 UNITS AND ENDURING UNDERSTANDINGS

Unit 1: Linear Functions and Linear Equations	Unit 2: Systems of Linear Equations and Inequalities
▪ A linear relationship has a constant rate of change, which can be visualized as the slope of the associated graph. ▪ There are many ways to algebraically represent a linear function and each form reveals different aspects of the function. ▪ Linear functions can be used to model contextual scenarios that involve a constant rate of change or data whose general trend is linear. ▪ A solution to a two-variable linear equation or inequality is an ordered pair that makes the equation or inequality true.	▪ A solution to a system of linear equations or inequalities is an ordered pair of numbers that satisfies all the equations or inequalities simultaneously. ▪ Solving a system of linear equations or inequalities is a process of determining the value or values that make the equation or inequality true. ▪ Systems of linear equations or inequalities can be used to model scenarios that include multiple constraints, such as resource limitations, goals, comparisons, and tolerances.
Unit 3: Quadratic Functions	**Unit 4: Exponent Properties and Exponential Functions**
▪ Quadratic functions have a linear rate of change. ▪ Quadratic functions can be expressed as a product of linear factors. ▪ Quadratic functions can be used to model scenarios that involve a linear rate of change and symmetry around a unique minimum or maximum. ▪ Every quadratic equation, $ax^2+bx+c=0$, where a is not zero, has at most two real solutions. These solutions can be determined using the quadratic formula.	▪ Properties of exponents are derived from the properties of multiplication and division. ▪ An exponential function has constant multiplicative growth or decay. ▪ Exponential functions can be used to model contextual scenarios that involve constant multiplicative growth or decay. ▪ Graphs and tables can be used to estimate the solution to an equation that involves exponential expressions.

Unit 1: Linear Functions and Linear Equations

Suggested Timing: Approximately 9 weeks

Linear relationships are among the most prevalent and useful relationships in mathematics and the real world. Any equality in two variables that exhibits a constant rate of change for these variables is linear. Real-world contexts that have a constant rate of change and data sets with a nearly constant rate of change can be effectively modeled by a linear function. Students explore all aspects of linear relationships in this unit: contextual problems that involve constant rate of change, lines in the coordinate plane, arithmetic sequences, and algebraic means of expressing a linear relationship between two quantities. Through this unit, students develop deep skills with linear functions and equations and an appreciation for the simplicity and power of linear functions as building blocks of all higher mathematics.

ENDURING UNDERSTANDINGS

Students will understand that ...

- A linear relationship has a constant rate of change, which can be visualized as the slope of the associated graph.
- There are many ways to algebraically represent a linear function and each form reveals different aspects of the function.
- Linear functions can be used to model contextual scenarios that involve a constant rate of change or data whose general trend is linear.
- A solution to a two-variable linear equation or inequality is an ordered pair that makes the equation or inequality true.

KEY CONCEPTS

- 1.1: Constant rate of change and slope
- 1.2: Linear functions
- 1.3: Linear equations
- 1.4: Linear models of nonlinear scenarios
- 1.5: Two-variable linear inequalities

KEY CONCEPT 1.1: CONSTANT RATE OF CHANGE AND SLOPE

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
1.1.1 Determine whether two quantities vary directly given a relationship represented graphically, numerically, algebraically, or verbally.	**1.1.1a** The graph of a direct variation whose domain is all real numbers is a non-vertical and non-horizontal line that contains the origin. **1.1.1b** Direct variation is a special case of a linear relationship where one quantity is proportional to another quantity. Two quantities vary directly if the ratio $\frac{y}{x}$ is constant for all (x, y) pairs. **1.1.1c** A direct variation can be expressed in the algebraic form $y = kx$, where k is a non-zero constant.
1.1.2 Calculate the constant rate of change of a linear relationship.	**1.1.2a** The constant rate of change of a linear relationship is the slope of the line of its associated graph. **1.1.2b** The constant rate of change of a linear relationship, whose associated line is non-vertical, can be graphically interpreted as the ratio of the vertical change of the line to the corresponding horizontal change of the line. **1.1.2c** The constant rate of change of a linear relationship can be calculated by finding the ratio of the change in the output to the change in the input using any two distinct ordered pairs and the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. **1.1.2d** Rate of change describes how two quantities change together. The unit for rate of change is the unit of the output variable per the unit of the input variable.
1.1.3 Create a graphical or numerical representation of a linear relationship given its constant rate of change.	**1.1.3a** Given any point, the slope of a line can be used to generate all points on the graph of the line that passes through the point. **1.1.3b** Given any initial condition, the constant rate of change of a linear relationship can be used to generate all other pairs of values that satisfy the relationship. **1.1.3c** If the relationship represented in a table of values has a constant rate of change, then the points on the associated graph will lie on a line. **1.1.3d** If the output values of a linear function differ by m when the input values differ by 1, then the output values differ by mk when the input values differ by k, where k is a real number.

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
1.1.4 Determine whether a relationship presented graphically or numerically is linear by examining the rate of change.	**1.1.4a** In a graph of ordered pairs that are linearly related, the ratio of the vertical change between any two points to the corresponding horizontal change between the same two points is constant. **1.1.4b** In a table of linearly related values where successive input values differ by a constant amount (e.g., differ by 1), successive output values will also differ by a constant amount. **1.1.4c** In a table of linearly related values where the input values differ by varying amounts, the associated output values will differ proportionally to these varying amounts.

Content Boundary: Direct variation is an extension of reasoning with proportional relationships, which students explored extensively in middle school. Students will have already solved context-free proportions in prior grades. Here, the focus is on analyzing the proportional nature of the relationship and using it to solve real-world problems.

Cross Connection: Students may be familiar with the slope formula from their previous courses. The focus here is on developing a thorough understanding of the rate of change of a linear relationship.

KEY CONCEPT 1.2: LINEAR FUNCTIONS

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
1.2.1 Determine whether a relationship is linear or nonlinear based on a numerical sequence whose indices increase by a constant amount.	**1.2.1a** An arithmetic sequence is a linear relationship whose domain consists of consecutive integers. **1.2.1b** The differences between successive terms of an arithmetic sequence are equal. **1.2.1c** An arithmetic sequence can be determined using the constant difference and any term in the sequence.
1.2.2 Convert a given representation of an arithmetic sequence to another representation of the arithmetic sequence.	**1.2.2a** The graph of an arithmetic sequence is a set of discrete points that lie on a line. **1.2.2b** Successive terms in an arithmetic sequence are obtained by adding the common difference to the previous term. To find the value of the term that occurs *n* terms after a specified term, add the common difference *n* times to the term. **1.2.2c** An arithmetic sequence can be algebraically expressed with the formula $a_n = a_k + d(n - k)$ where a_n is the *n*th term, a_k is the *k*th term, and *d* is the constant difference between successive terms. **1.2.2d** A verbal representation of an arithmetic sequence describes a discrete domain and a constant rate of change.
1.2.3 Use function notation to describe the relationship between an input–output pair of a function.	**1.2.3a** A function is a type of relationship between two quantities where each input is related to one (and only one) value of the output. **1.2.3b** The domain of a function is the set of all inputs for the function. The range of a function is the set of all outputs for the function resulting from the set of inputs. **1.2.3c** The notation "$f(x)$" is read as "*f* of *x*"; "*f*" is the name of a function, "*x*" stands for any input value in the domain of the function, and "$f(x)$" represents the output value in the range of the function that corresponds to the input value. **1.2.3d** Any solution (x, y) to the equation $y = f(x)$ represents a point that lies on the graph of function *f*.

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
1.2.4 Convert a given representation of a linear function to another representation of the linear function.	**1.2.4a** A graphical representation of a linear function displays ordered pairs satisfying the relationship. The exact coordinates of the ordered pairs may or may not be evident from the graph of the function. **1.2.4b** A numerical representation of a linear function usually consists of only a subset of the ordered pairs that satisfy the relationship. Any two distinct ordered pairs can be used to generate a graph of the relationship and compute the constant rate of change. **1.2.4c** An algebraic representation of a linear function contains the complete information about the function because any output value can be determined from a given input value. **1.2.4d** A verbal representation of a linear function describes the constant rate of change and known values of the function.
1.2.5 Translate between algebraic forms of a linear function, using purposeful algebraic manipulation.	**1.2.5a** Common algebraic forms of linear functions include point–slope form, $y = y_1 + m(x - x_1)$, and slope–intercept form, $y = mx + b$. **1.2.5b** Slope–intercept form is a special case of point–slope form where $(x_1, y_1) = (0, b)$. **1.2.5c** Purposeful algebraic manipulation can reveal information about how the quantities in a linear function relate to each other.
1.2.6 Model a contextual scenario with a linear function.	**1.2.6a** Linear functions can be used to model contextual scenarios that involve a constant rate of change of a dependent variable (the output) with respect to an independent variable (the input). **1.2.6b** A linear function derived from a contextual scenario can be solved free of context, but the solution must be interpreted in context to be correctly understood. **1.2.6c** Two distinct input–output pairs from a contextual scenario that involves a constant rate of change can be used to determine a linear function that models the scenario. **1.2.6d** A constant rate of change and a corresponding input–output pair from a contextual scenario can be used to determine a linear function that models the scenario.

Content Boundary: Students will use arithmetic sequences to help them understand linear functions. By the end of the unit, students should understand that a sequence is a function with whole-number inputs, however knowing formulas associated with arithmetic sequences is beyond the scope of this course.

Cross Connection: Students will come to Algebra 1 with some prior knowledge about linear relationships. However, this knowledge might be procedural (e.g., how to calculate slope) or fragmented (e.g., not connecting the value of b in $y = mx + b$ with the y-intercept of a line). This course guides students to consolidate and make connections among the disparate pieces of information they have relating to linear functions by understanding that a constant rate of change is the defining feature of a linear relationship.

KEY CONCEPT 1.3: LINEAR EQUATIONS

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
1.3.1 Convert a given representation of a linear equation to another representation of that linear equation.	**1.3.1a** A graphical representation of a linear equation is a set of ordered pairs that satisfy the relationship. The exact coordinates of the ordered pairs may or may not be evident from the graph of the equation. **1.3.1b** A numerical representation of a linear equation consists of only a subset of the ordered pairs that satisfy the relationship. It can be used to generate a graph of the relationship. **1.3.1c** An algebraic representation of a linear equation often takes the form $Ax + By = C$, where the parameters *A*, *B*, and *C* are non-zero constants. This form is called the standard form of a linear equation. **1.3.1d** A linear equation in two variables can be used to represent contextual scenarios where there exists a constraint or condition on the variables and neither variable is necessarily considered an input or output.
1.3.2 Interpret the solutions to a two-variable linear equation.	**1.3.2a** A solution to a linear equation, $Ax + By = C$, is an ordered pair (x, y) that makes the equation true. **1.3.2b** A linear equation derived from a contextual scenario can be solved free of context, but the solution must be interpreted in context to be correctly understood. **1.3.2c** The solution to a linear equation derived from a contextual scenario should use the same units as the variables in the contextual scenario.
1.3.3 Rewrite a two-variable linear equation in terms of one of the variables to preserve the solution set, using purposeful algebraic manipulation.	**1.3.3a** The solution set to a linear equation, $Ax + By = C$, is the set of all ordered pairs (x, y) that make the equation true. **1.3.3b** Purposeful algebraic manipulation can reveal information about how the quantities in a linear equation relate to each other.
1.3.4 Construct representations of parallel or perpendicular lines.	**1.3.4a** The slopes of parallel lines are equal, and two distinct lines with equal slopes are parallel. **1.3.4b** The slopes of non-vertical and non-horizontal perpendicular lines are multiplicative inverses of each other with opposite signs. **1.3.4c** A vertical line is perpendicular to a horizontal line, and vice versa. **1.3.4d** An equation for a line parallel or perpendicular to a given line can be determined using the slope of the given line and a point not on the given line.

Cross Connection: In this key concept, students regard the two variables in a linear equation as two independent quantities related by a constraint; this is distinct from the input–output thinking that characterized the relationships between the quantities in the previous key concept. Students should make connections with the one-variable equations they solved in middle school, understanding that both one-variable and two-variable equations are statements that can be either true or false.

KEY CONCEPT 1.4: LINEAR MODELS OF NONLINEAR SCENARIOS

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
1.4.1 Interpret a graphical representation of a piecewise linear function in context.	**1.4.1a** A piecewise linear function consists of two or more linear functions, each restricted to nonoverlapping intervals of input values.
1.4.2 Construct a graphical representation of a piecewise linear function to model a contextual scenario.	**1.4.2a** A contextual scenario that involves different constant rates of change over different intervals of the domain can be modeled using a piecewise linear function. **1.4.2b** A contextual scenario that involves a constant rate of change where the input or output values do not change continuously can be modeled using a piecewise linear function.
1.4.3 Determine whether the scatterplot of the relationship between two quantities can be reasonably modeled by a linear model.	**1.4.3a** A scatterplot whose points fall roughly in the shape of an ellipse can often be modeled usefully by a linear equation. **1.4.3b** Sets of data that show a graphically upward trend (as the input value increases) are said to have a positive association. **1.4.3c** Sets of data that show a graphically downward trend (as the input value increases) are said to have a negative association.
1.4.4 Determine an equation for a trend line that describes trends in a scatterplot.	**1.4.4a** A trend line describes an observed relationship between the variables in a scatterplot but may or may not contain any of the data points. **1.4.4b** A trend line does not perfectly model the data, so values predicted using the model can be expected to differ from actual values.
1.4.5 Use an equation for a trend line to predict values in context.	**1.4.5a** The equation for a trend line can be used to estimate either the input or output quantities in context. **1.4.5b** Relationships derived from data usually have limited domains beyond which the trend line might become an increasingly poor model.

Content Boundary: Students should explore piecewise linear graphs that model scenarios that have a variety of constant rates of change over different intervals. Writing a single function expression for a piecewise function with multiple linear functions, such as $f(x)=\begin{cases}2x-5, x<0\\-3x+6, x\geq 0\end{cases}$, is beyond the scope of this course. Engaging with graphs of piecewise functions that have nonlinear components is also beyond the scope of this course.

Content Boundary: Students should be able to determine if a linear model is appropriate given a scatterplot and to make and justify reasonable choices about how to construct a line that fits the data. Students could calculate the residuals of their line as one way to measure the appropriateness of fit, but doing so is beyond the scope of this course. Calculating a regression equation, either by hand or with technology, is beyond the scope of this course and should be reserved for Algebra 2 or beyond. The emphasis is on using (as opposed to constructing) the linear function model for the data.

Cross Connection: As AP Statistics students will learn, the equation for a trend line calculated from a data set can be used to predict both input and output values for the relationship that is modeled by that trend line. However, it is not appropriate to use a regression equation to predict input values from output values. This is because the mathematics behind the least squares regression assumes that the input values are fixed and a line is fitted to predict the output values given the input values.

KEY CONCEPT 1.5: TWO-VARIABLE LINEAR INEQUALITIES

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
1.5.1 Convert a given representation of a linear inequality to another representation of the linear inequality.	**1.5.1a** A graphical representation of a linear inequality is a set of ordered pairs that satisfy the relationship. The exact coordinates of the ordered pairs may or may not be evident from the graph of the inequality. **1.5.1b** A numerical representation of a linear inequality usually consists of only a subset of the ordered pairs that satisfy the relationship. It can be used to generate a graph of the relationship. **1.5.1c** An algebraic representation of a linear inequality usually relates the expressions $Ax+By$ and C, where the parameters A, B, and C are non-zero constants, with an inequality symbol, $<$, $\leq$, $>$, or $\geq$. **1.5.1d** A linear inequality is useful for modeling contextual scenarios that include resource limitations, goals, constraints, comparisons, and tolerances.
1.5.2 Determine solutions to a two-variable inequality.	**1.5.2a** A solution to an inequality in two variables is an ordered pair that makes the inequality true. **1.5.2b** The solution set to a two-variable inequality can be displayed graphically by a half-plane. Any coordinate in the half-plane, or on its boundary if the boundary is included, is a solution to the inequality. **1.5.2c** A solution to a two-variable linear inequality that represents a contextual scenario is a pair of numbers that satisfies the constraints of the contextual scenario.
1.5.3 Rewrite a linear inequality in terms of one of the variables to preserve the solution set, using purposeful algebraic manipulation.	**1.5.3a** The solution set to an inequality in two variables is the set of all ordered pairs that make the inequality true. **1.5.3b** Adding the same real number to or subtracting the same real number from both sides of an inequality does not change the inequality relationship. **1.5.3c** Multiplying both sides of an inequality by the same positive real number or dividing both sides of an inequality by the same positive real number does not change the inequality relationship. **1.5.3d** Multiplying both sides of an inequality by the same negative real number or dividing both sides of an inequality by the same negative real number reverses the direction of the inequality relationship.

Content Boundary: Applications of two-variable linear inequalities are beyond the scope of this unit. They are addressed at the end of Unit 2: Systems of Linear Equations and Inequalities. This key concept focuses on determining if a given ordered pair is a solution to a linear inequality and graphing the solution set on a coordinate plane.

Unit 2: Systems of Linear Equations and Inequalities

Suggested Timing: Approximately 5 weeks

Across Unit 2, students are asked to solve systems of equations in support of two goals: to determine the solution to the system of equations and to become strategic and efficient in choosing a method to solve the system. Students use systems of linear equations and systems of linear inequalities to model physical phenomena, especially those with multiple constraints where an optimal solution to an objective function is desired. Through these contexts students build upon their prior knowledge of solving systems of equations and develop more sophisticated understandings about what the solution(s) to a system means in the context of the problem.

ENDURING UNDERSTANDINGS

Students will understand that …

- A solution to a system of linear equations or inequalities is an ordered pair of numbers that satisfies all the equations or inequalities simultaneously.
- Solving a system of linear equations or inequalities is a process of determining the value or values that make the equation or inequality true.
- Systems of linear equations or inequalities can be used to model scenarios that include multiple constraints, such as resource limitations, goals, comparisons, and tolerances.

KEY CONCEPTS

- 2.1: The solution to a system of equations
- 2.2: Solving a system of linear equations algebraically
- 2.3: Modeling with systems of linear equations
- 2.4: Systems of linear inequalities

KEY CONCEPT 2.1: THE SOLUTION TO A SYSTEM OF EQUATIONS

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
2.1.1 Use a graph or tables of values to estimate the solution to a system of equations.	**2.1.1a** A solution to a system of linear equations, if one exists, is an intersection point of the lines corresponding to the equations. **2.1.1b** A solution to a system of linear equations, if one exists, corresponds to the solutions that the equations have in common.
2.1.2 Determine the number of real solutions to a system of two linear equations.	**2.1.2a** A system of two linear equations can have no solution, one solution, or infinitely many solutions. **2.1.2b** If the graphs of two linear equations in a system are parallel, then the system has no solutions. If the graphs of two linear equations are not parallel and do not coincide, then the system has one solution. If the graphs of two linear equations coincide, then the system has infinitely many solutions.
2.1.3 Determine the intersection point(s) of the graphs of two functions *f* and *g*.	**2.1.3a** The graphs of functions *f* and *g* intersect at $x = k$ if $f(k) = g(k)$. **2.1.3b** The graphs of the functions *f* and *g* intersect at $x = k$ if $f(k) - g(k) = 0$.

Content Boundary: In this unit, students will work with systems of two linear equations in two variables. In Unit 3: Quadratic Functions, students will be exposed to systems of one quadratic and one linear equation and to systems of two quadratic equations. Systems of three linear equations in three variables are beyond the scope of this course.

KEY CONCEPT 2.2: SOLVING A SYSTEM OF LINEAR EQUATIONS ALGEBRAICALLY

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
2.2.1 Solve a system of linear equations using algebraic methods.	**2.2.1a** Algebraic methods of solving a system of equations include the substitution method and the elimination method. **2.2.1b** An efficient method of solving a system of equations should be based on the forms of the equations in the system.
2.2.2 Justify the steps used to algebraically solve a system of linear equations.	**2.2.2a** Algebraically equivalent expressions can be used interchangeably in equations. **2.2.2b** In a system of equations, substituting one equation with a multiple of that equation or substituting one equation with the sum of non-zero multiples of the equations will result in a system with the same solutions as the original system.

KEY CONCEPT 2.3: MODELING WITH SYSTEMS OF LINEAR EQUATIONS

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
2.3.1 Model a contextual scenario with a system of linear equations.	**2.3.1a** A system of linear equations can be used to determine when two linear functions that model a contextual scenario have the same input–output pair. **2.3.1b** A system of linear equations can be used to model a contextual scenario in which two quantities are subject to multiple constraints. **2.3.1c** A system of linear equations derived from a contextual scenario can be solved free of context, but the solution must be interpreted in context to be correctly understood. **2.3.1d** The solution to a system of linear equations used to model a contextual scenario should use the same units as the variables in the contextual scenario.
2.3.2 Use the rates of change to draw conclusions about a contextual scenario modeled by a system of linear equations.	**2.3.2a** If the multiple constraints in a contextual scenario modeled by a system of linear equations each have a different constant rate of change, then the scenario will have one solution. The graph associated with the system will consist of a pair of intersecting lines. **2.3.2b** If the multiple constraints in a contextual scenario modeled by a system of linear equations each have the same constant rate of change and the linear equations in the system share at least one solution, then the scenario will have infinitely many solutions. The graph associated with the system will consist of a pair of coinciding lines. **2.3.2c** If the multiple constraints in a contextual scenario modeled by a system of linear equations each have the same constant rate of change and the linear equations in the system differ in at least one solution, then the scenario will have no solution. The graph associated with the system will consist of a pair of parallel lines.

Content Boundary: The focus of this key concept is to become strategic and efficient about choosing a particular method to solve a system of linear equations. Students should come to understand that the graphical and tabular methods of solving a system of equations are inefficient and imprecise in most cases. Students should appreciate that the elimination method is convenient for linear equations written in standard form. However, elimination is not generally applicable for nonlinear systems. The substitution method has wider utility for nonlinear systems. In this unit, students will only explore linear systems, but they need to develop fluency with the substitution method because it will become more important later in the course and in future courses. Other algebraic techniques, such as those involving matrices, are beyond the scope of this course.

KEY CONCEPT 2.4: SYSTEMS OF LINEAR INEQUALITIES

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
2.4.1 Use algebra to determine if an ordered pair is a solution to a system of linear inequalities.	**2.4.1a** If an ordered pair is a solution to a system of inequalities, it will make all inequalities that constitute the system true.
2.4.2 Graphically represent the solution to a system of two-variable inequalities.	**2.4.2a** The solution to a system of linear inequalities is the intersection of half-planes that correspond to the individual inequalities in the system. Every point located in the solution region, or on the boundary if the boundary is included, is a solution to the system.
2.4.3 Model a contextual scenario with a system of linear inequalities.	**2.4.3a** A system of linear inequalities can be used to model a contextual scenario in which the relationship between two quantities is subject to multiple constraints, such as resource limitations, goals, comparisons, and tolerances. **2.4.3b** A solution, if one exists, to a system of inequalities used to model a contextual scenario is a set of values that satisfies the constraints of the scenario. **2.4.3c** A system of linear inequalities that models a contextual scenario can be solved free of context, but any solution must be interpreted in context to be correctly understood. **2.4.3d** A solution to a system of linear inequalities used to model a contextual scenario should involve the same units as the variables in the contextual scenario.

Cross Connection: Students should make connections with their knowledge from the previous unit and prior courses to understand that any situation that involves an inequality is often best represented by a graphical representation. That is, the solution to a one-variable linear inequality can be displayed as a shaded portion of a number line, the solution to a two-variable linear inequality can be displayed as a half-plane, and the solution to a system of two-variable linear inequalities can be displayed as the intersection of the associated half-planes.

Content Boundary: Systems of inequalities that involve nonlinear functions, such as quadratics or other polynomials, are beyond the scope of this course.

Unit 3: Quadratic Functions

Suggested Timing: Approximately 9 weeks

In this unit, students develop a strong foundation in the important concept of quadratic functions. Students should understand that quadratic functions have a linear rate of change and are often formed by multiplying two linear expressions, and therefore are not linear. Quadratic functions are useful for modeling phenomena that have a linear rate of change and symmetry around a unique minimum or maximum. This foundational understanding of quadratics helps students build their conceptual knowledge of nonlinear functions and prepares them for further study of polynomial and rational functions in Algebra 2.

ENDURING UNDERSTANDINGS

Students will understand that …

- Quadratic functions have a linear rate of change.
- Quadratic functions can be expressed as a product of linear factors.
- Quadratic functions can be used to model scenarios that involve a linear rate of change and symmetry around a unique minimum or maximum.
- Every quadratic equation, $ax^2 + bx + c = 0$, where a is not zero, has at most two real solutions. These solutions can be determined using the quadratic formula.

KEY CONCEPTS

- 3.1: Functions with a linear rate of change
- 3.2: The algebra and geometry of quadratic functions
- 3.3: Solving quadratic equations
- 3.4: Modeling with quadratic functions

KEY CONCEPT 3.1: FUNCTIONS WITH A LINEAR RATE OF CHANGE

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
3.1.1 Determine whether a relationship is quadratic or nonquadratic based on a numerical sequence whose indices increase by a constant amount.	**3.1.1a** In a table of values that represents a quadratic relationship and that has constant step sizes, the differences in the values of the relationship, called the first differences, exhibit a linear pattern. The second differences of a quadratic sequence are constant. **3.1.1b** Successive terms in a quadratic sequence can be obtained by adding corresponding successive terms of an arithmetic sequence.
3.1.2 Convert a given representation of a quadratic function to another representation of the quadratic function.	**3.1.2a** A graphical representation of a quadratic function displays ordered pairs that satisfy the relationship. The exact coordinates of the ordered pairs may or may not be evident from the graph of the function. **3.1.2b** A numerical representation of a quadratic function consists of only a subset of the ordered pairs that satisfy the relationship. **3.1.2c** An algebraic form of a quadratic function contains the complete information about the function because any output value can be determined from any given input value. **3.1.2d** A verbal representation of a quadratic function could refer to a symmetric scenario with a unique minimum or maximum.

Cross Connection: This key concept motivates the need for a nonlinear function to represent scenarios that cannot be adequately modeled by a linear function, such as gravity or area. Students should understand that any scenario, physical or otherwise, that involves the product of two linear expressions will be quadratic. Earlier in Algebra 1, students saw that physical scenarios that exhibit a constant rate of change can be modeled by a linear function. In AP Calculus and AP Physics, students will come to understand that particle motion problems that exhibit a constant acceleration can be modeled by a quadratic function.

KEY CONCEPT 3.2: THE ALGEBRA AND GEOMETRY OF QUADRATIC FUNCTIONS

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
3.2.1 Identify key characteristics of the graph of a quadratic function.	**3.2.1a** The graph of a quadratic function is a parabola. The parabola is symmetric about a vertical line that passes through the vertex of the parabola. **3.2.1b** The vertex of a parabola is the point on the curve where the outputs of the function change from increasing to decreasing or vice versa. The y-coordinate of the vertex of a parabola is the maximum or minimum value of the function. **3.2.1c** A parabola can have two x-intercepts, one x-intercept, or no x-intercepts. **3.2.1d** If the vertex of a parabola is a minimum value, then the parabola is said to be concave up. If the vertex is a maximum value, then the parabola is said to be concave down.
3.2.2 Translate between algebraic forms of a quadratic function using purposeful algebraic manipulation.	**3.2.2a** Common algebraic forms of a quadratic function include standard form, $f(x)=ax^2+bx+c$; factored form, $f\left(x\right)=a\left(x-r\right)\left(x-s\right)$; and vertex form, $f(x)=a(x-h)^2+k$, where a is not zero. **3.2.2b** Every quadratic function has a standard form and a vertex form, but not all quadratic functions have a factored form over the real numbers. **3.2.2c** The standard form can be purposefully manipulated into the vertex form of the same quadratic function by completing the square. **3.2.2d** The standard form and the factored form of a quadratic function can be translated into each other with purposeful use of the distributive property.

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
3.2.3 Describe key features of the graph of a quadratic function in reference to an algebraic form of the quadratic function.	**3.2.3a** The graph of a quadratic function whose standard form is $f(x)=ax^2+bx+c$ has a vertex with the x-coordinate at $x=-\frac{b}{2a}$. The y-coordinate of the vertex can be calculated by evaluating the function rule using the x-coordinate of the vertex. The graph is symmetric about the vertical line $x=-\frac{b}{2a}$. **3.2.3b** The graph of a quadratic function whose factored form is $f(x)=a(x-r)(x-s)$, where $a\neq 0$ and $r\neq s$, has two x-intercepts, at $(r,0)$ and $(s,0)$. **3.2.3c** The graph of a quadratic function whose vertex form is $f(x)=a(x-h)^2+k$ has a vertex at the coordinate (h,k). **3.2.3d** The graph of a quadratic function of the form $f(x)=a(x-r)^2$, where $a\neq 0$, can be interpreted as the factored form where the parabola has one x-intercept at $(r,0)$ or as the vertex form where the vertex of the parabola is at $(r,0)$.
3.2.4 Determine an algebraic rule for a quadratic function given a sufficient number of points from the graph.	**3.2.4a** There is a unique parabola that includes any three distinct noncollinear points. A quadratic function whose graph is the parabola that contains these points can be determined using their coordinates. **3.2.4b** If two distinct inputs of a quadratic function are associated with equal outputs, then the x-coordinate of the vertex of the parabola is located halfway between these inputs. **3.2.4c** The x-intercepts of a quadratic function are two convenient points that, with a third point, can be used to determine an algebraic rule for the quadratic function. **3.2.4d** An algebraic rule for a quadratic function can be determined from the vertex and one other point on the graph of the parabola.

Content Boundary: Quadratic functions can be written in many forms, and each form reveals or obscures certain features of the quadratic. Students should be able to translate between these forms and choose the representation that best serves the problem at hand. Fluent skill in transforming one algebraic form into another will help students in Algebra 2 and beyond in investigating the structure of polynomial and rational functions. Rote exercises in transforming expressions are less effective at producing fluency than exercises in which the student purposefully transforms expressions for a specific reason.

It is possible to orient a parabola so it opens to the left or right. However, the associated equation would not be a function of x and is beyond the scope of this course.

KEY CONCEPT 3.3: SOLVING QUADRATIC EQUATIONS

Learning Objectives *Students will be able to …*	**Essential Knowledge** *Students need to know that …*
3.3.1 Describe the relationship between the algebraic and graphical representations of a quadratic equation.	**3.3.1a** The solutions to the quadratic equation $ax^2+bx+c=d$ can be interpreted graphically as the point(s) of intersection between the parabola defined by $y=ax^2+bx+c$ and the horizontal line defined by $y=d$. **3.3.1b** If a quadratic equation $ax^2+bx+c=0$ has real solutions $x=r$ and $x=s$, then the parabola defined by $y=ax^2+bx+c$ has x-intercepts at $(r,0)$ and $(s,0)$.
3.3.2 Solve quadratic equations by taking a square root.	**3.3.2a** For any positive real number a, there are two real numbers that satisfy the equation $x^2=a$, one positive and one negative. **3.3.2b** The notation "$\sqrt{a}$" represents the square root of a and refers only to the principal square root, or non-negative, number whose square equals a. **3.3.2c** There is no real number that will satisfy the equation $x^2=a$, when a is a negative real number.
3.3.3 Solve quadratic equations by factoring.	**3.3.3a** Factoring a quadratic expression yields an equivalent form of the expression that can be used to determine the roots of the associated quadratic equation. **3.3.3b** There can be multiple correct factorizations of a quadratic expression. **3.3.3c** The solutions of a quadratic equation in the form $a(x-r)(x-s)=0$ are $x=r$ and $x=s$ when $a\neq 0$.
3.3.4 Solve quadratic equations by completing the square.	**3.3.4a** Completing the square is an algebraic process of transforming a quadratic expression into a form that can be solved by adding, multiplying, and taking the square root. **3.3.4b** Every quadratic equation can be solved by completing the square, but the solutions may not be real numbers.

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
3.3.5 Solve quadratic equations using the quadratic formula.	**3.3.5a** Any quadratic equation can be written in the form $mx^2+nx+p=q$, which can be purposefully manipulated into the standard form of a quadratic equation, $ax^2+bx+c=0$. **3.3.5b** The quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, can be used to solve any quadratic equation of the form $ax^2+bx+c=0$, but the solutions may not be real numbers. **3.3.5c** The quadratic formula can be derived by completing the square on the standard form of a quadratic equation. **3.3.5d** The quadratic formula can be written as the sum of two terms, $x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$, which shows that the x-intercepts are each a horizontal distance of $\frac{\sqrt{b^2-4ac}}{2a}$ from the x-coordinate of the vertex.
3.3.6 Determine the number of real solutions to a quadratic equation.	**3.3.6a** If a quadratic equation has two rational solutions, then it can be factored into linear factors with integer coefficients. **3.3.6b** Given a quadratic equation of the form $ax^2+bx+c=0$, the value of the discriminant of the quadratic equation ($D=b^2-4ac$) can be used to determine whether the quadratic equation has two distinct real solutions ($D>0$), one real solution ($D=0$), or no real solutions ($D<0$).

Content Boundary: A focus of this key concept is having students make explicit connections between the real number solutions of a quadratic equation and the x-intercept(s) of the associated graph. Students should understand that all quadratic equations can be solved, but some quadratic equations require a new number system to adequately express the solution set. However, imaginary numbers are beyond the scope of this course.

KEY CONCEPT 3.4: MODELING WITH QUADRATIC FUNCTIONS

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
3.4.1 Model a contextual scenario with a quadratic function.	**3.4.1a** A contextual scenario where the output quantity increases and then decreases (or vice versa), such as accelerated motion, can be effectively modeled by a quadratic function. **3.4.1b** A contextual scenario whose physical manifestation resembles a parabola, such as a satellite dish or solar collector, can be effectively modeled by a quadratic function. **3.4.1c** A contextual scenario that can be expressed as a product of two linear expressions, such as profit or area, can be effectively modeled by a quadratic function.
3.4.2 Interpret solutions to quadratic equations derived from contextual scenarios.	**3.4.2a** A quadratic equation derived from a contextual scenario can be solved free of context, but the solution must be interpreted in context to be correctly understood. **3.4.2b** The solution to a quadratic equation derived from a context should involve the same units as the variables in the contextual scenario.
3.4.3 Interpret the vertex and roots of a quadratic model in context.	**3.4.3a** If the values of the vertex are included in the contextual domain and range of the problem, the x-value of the vertex of the parabola represents the input value that corresponds to either the minimum or maximum output value, and the y-value of the vertex of the parabola represents the minimum or maximum output value. **3.4.3b** The x-value(s) of the root(s) of a parabola often represent the extreme values (of the input variable) in a contextual scenario.

Unit 4: Exponent Properties and Exponential Functions

Suggested Timing: Approximately 5 weeks

Students explore exponent rules as an extension of geometric sequences and the properties of multiplication and division for real numbers. Students should make sense of exponent rules and not simply memorize them without understanding how they arise. The unit culminates in students investigating how exponential functions can model physical phenomena that exhibit a constant multiplicative growth. Exponential functions are framed as multiplicative analogues of linear functions. Thus, a tight connection should be drawn between these two classes of functions and their shared properties.

ENDURING UNDERSTANDINGS

Students will understand that ...

- Properties of exponents are derived from the properties of multiplication and division.
- An exponential function has constant multiplicative growth or decay.
- Exponential functions can be used to model contextual scenarios that involve constant multiplicative growth or decay.
- Graphs and tables can be used to estimate the solution to an equation that involves exponential expressions.

KEY CONCEPTS

- 4.1: Exponent rules and properties
- 4.2: Roots of real numbers
- 4.3: Sequences with multiplicative patterns
- 4.4: Exponential growth and decay

KEY CONCEPT 4.1: EXPONENT RULES AND PROPERTIES

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
4.1.1 Use exponent rules to express products and quotients of exponential expressions in equivalent forms.	**4.1.1a** Exponential expressions involving multiplication can be rewritten by invoking the rule $n^a \bullet n^b = n^{a+b}$, where $n > 0$. **4.1.1b** Exponential expressions involving division can be rewritten by invoking the rule $\frac{n^a}{n^b} = n^{a-b}$, where $n > 0$. **4.1.1c** Exponential expressions involving powers of powers can be rewritten by invoking the rule $\left(n^a\right)^b = n^{ab}$, where $n > 0$.
4.1.2 Use exponent rules to express numerical and variable expressions that involve negative exponents using positive exponents, and vice versa.	**4.1.2a** Any non-zero real number raised to the zero power is equal to 1. That is, $n^0 = 1$, where $n \neq 0$. **4.1.2b** Zero raised to the zero power is not defined in the real number system. **4.1.2c** A negative exponent of −1 can be used to represent a reciprocal. That is, $n^{-k} = \frac{1}{n^k}$, where $n \neq 0$. **4.1.2d** The properties of negative integer exponents, and those of an exponent of zero, are extensions of the properties of positive integer exponents.

Content Boundary: Students coming to Algebra 1 may be familiar with negative exponents from working with scientific notation in earlier courses, but their skills could be limited to superficial knowledge like "moving the decimal point." The focus here is for students to develop these formulas through extending the properties of multiplication and division. Students should not be expected to simplify excessively complicated quotients, such as $\frac{36x^3y^{-4}z^7}{72x^{-5}y^6z^8}$, because these expressions have limited usefulness outside of rote skill acquisition.

Cross Connection: Students should be able to flexibly and fluently translate among different forms of expressions involving exponents. For example, often in AP Calculus it is advantageous to rewrite the function $f\left(x\right) = \frac{1}{x^2}$ as $f\left(x\right) = x^{-2}$.

KEY CONCEPT 4.2: ROOTS OF REAL NUMBERS

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
4.2.1 Perform operations with rational and irrational numbers.	**4.2.1a** The value of an irrational number cannot be expressed exactly as a ratio of integers or as a nonrepeating, nonterminating decimal, and is often represented exactly by a symbol such as π or $\sqrt{2}$. **4.2.1b** An irrational number can be approximated to any specified degree of precision by a rational number.
4.2.2 Represent square roots of real numbers in equivalent forms.	**4.2.2a** The square root of a squared real number a is equivalent to the absolute value of the number. That is, $\sqrt{a^2} = \lvert a \rvert$. **4.2.2b** For any two nonnegative real numbers a and b, the product of their square roots is equal to the square root of their product. That is, $\sqrt{a} \bullet \sqrt{b} = \sqrt{ab}$. **4.2.2c** If a positive real number a can be written as the product of the square of a positive number b and another positive number c—that is, if $a = b^2c$—then the square root of a is equal to the square root of the product of b^2 and c. That is, $\sqrt{a} = \sqrt{b^2c} = \sqrt{b^2} \bullet \sqrt{c} = b\sqrt{c}$ where a, b, and c are all positive.
4.2.3 Use the laws of exponents to represent roots of real numbers in terms of rational number powers.	**4.2.3a** The square root of a nonnegative real number can be expressed with the exponent $\frac{1}{2}$. That is, $\sqrt{n} = n^{\frac{1}{2}}$ where $n \geq 0$. **4.2.3b** The cube root of any real number can be expressed with the exponent $\frac{1}{3}$. That is, $\sqrt[3]{n} = n^{\frac{1}{3}}$. **4.2.3c** The properties of rational exponents are extensions of the properties of integer exponents.

Content Boundary: The focus in this key concept is on having students understand that "simplifying" a square root is similar to reducing a fraction to lowest terms. That is, $\sqrt{8}$ expresses the same quantity as $2\sqrt{2}$, just as $\frac{6}{8}$ expresses the same quantity as $\frac{3}{4}$. It is not absolutely necessary to reduce a fraction to lowest terms and it is not absolutely necessary to simplify a square root to "lowest terms." Different equivalent forms of numbers exist, and the context of the problem may suggest when one form could be more useful than another.

Cross Connection: Students should start to identify circumstances where a particular form of a number provides an insight or advantage that other forms do not. In AP Calculus, students will often find it helpful to rewrite the function $f(x) = \sqrt{x}$ as $f(x) = x^{\frac{1}{2}}$.

KEY CONCEPT 4.3: SEQUENCES WITH MULTIPLICATIVE PATTERNS

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
4.3.1 Determine whether a relationship is exponential or nonexponential based on a numerical sequence whose indices increase by a constant amount.	**4.3.1a** A geometric sequence is an exponential relationship whose domain consists of consecutive integers. **4.3.1b** The ratios of successive terms of a geometric sequence are equivalent. **4.3.1c** A geometric sequence can be determined from the common ratio and any term in the sequence.
4.3.2 Convert a given representation of a geometric sequence to another representation of the geometric sequence.	**4.3.2a** The graph of a geometric sequence is a set of discrete points that lie on a curve. **4.3.2b** Successive terms in a geometric sequence are obtained by multiplying the previous term by the common ratio. To find the value of the term that occurs *n* terms after a specified term, multiply the specified term by the common ratio *n* times. **4.3.2c** A geometric sequence can be algebraically expressed with the formula $a_n = a_k \bullet c^{(n-k)}$, where a_n is the *n*th term, a_k is the *k*th term, and *c* is the common ratio between successive terms. **4.3.2d** A verbal representation of a geometric sequence describes a discrete domain and a constant multiplicative growth or decay.
4.3.3 Determine whether a relationship is exponential by analyzing a graphical, numerical, algebraic, or verbal representation.	**4.3.3a** A graph of an exponential function is a curve that exhibits asymptotic behavior to the left or right. **4.3.3b** The numerical representation of an exponential function will have ordered pairs where, if the inputs differ by a constant amount, then the ratios of corresponding outputs are equivalent. **4.3.3c** An algebraic representation of an exponential function often takes the form $f(x) = a \bullet c^x$, $c > 0$, where *c* is the constant growth or decay factor. **4.3.3d** A verbal representation of an exponential function describes a constant multiplicative growth or decay and known values from the relationship.

Cross Connection: The goal of introducing students to geometric sequences is to have them investigate multiplicative patterns as a counterpoint to the additive patterns of linear and quadratic sequences. Knowing formulas associated with geometric sequences is beyond the scope of this course.

KEY CONCEPT 4.4: EXPONENTIAL GROWTH AND DECAY

Learning Objectives *Students will be able to ...*	**Essential Knowledge** *Students need to know that ...*
4.4.1 Calculate a growth or decay factor of an exponential relationship.	**4.4.1a** In an exponential relationship, the output values grow by equal factors over equal intervals. **4.4.1b** Given two input–output pairs in an exponential relationship, $(a, f(a))$ and $(b, f(b))$, where b is n units more than a, the n-unit growth (or decay) factor is the quotient of the corresponding outputs, $\frac{f(b)}{f(a)}$. **4.4.1c** Given two input–output pairs in an exponential relationship, $(a, f(a))$ and $(b, f(b))$, if the quotient of any two outputs, $\frac{f(b)}{f(a)}$, where $b > a$, is greater than 1, then the value is called a growth factor. **4.4.1d** Given two input–output pairs in an exponential relationship, $(a, f(a))$ and $(b, f(b))$, if the quotient of any two outputs, $\frac{f(b)}{f(a)}$, where $b > a$, is between 0 and 1, then the value is called a decay factor.
4.4.2 Create graphical or numerical representations of an exponential function using the common growth or decay factor.	**4.4.2a** Given any point on the graph of an exponential function, the common growth or decay factor can be used to generate all points on the graph of the curve that contains the point. **4.4.2b** Given any input–output pair from an exponential function, the common growth or decay factor can be used to generate all other pairs of values that satisfy the relationship. **4.4.2c** If the relationship represented in a table of values has a common ratio in outputs over equal differences of inputs, then the points on the associated graph will lie on a curve that is asymptotic to the left or the right. **4.4.2d** If the output values change by a factor of c when the input values differ by 1, then the output values change by a factor of c^k when the input values differ by k, where k is a real number.

Learning Objectives *Students will be able to ...*	Essential Knowledge *Students need to know that ...*
4.4.3 Convert a given representation of an exponential function to another representation of the exponential function.	**4.4.3a** The graphical representation of an exponential function displays ordered pairs that satisfy the relationship. The exact coordinates of the ordered pairs may or may not be evident from the graph of the function. **4.4.3b** A numerical representation of an exponential function consists of only a subset of the ordered pairs that satisfy the relationship, but can be used to compute the common ratio of growth or decay and determine all other aspects of the exponential function. **4.4.3c** An algebraic representation of an exponential function contains the complete information about the function because any output value can be determined from any given input value. **4.4.3d** A verbal representation of an exponential function describes the common ratio of growth or decay and/or known input–output pairs from the function.
4.4.4 Approximate input and output values of an exponential function using representations of the exponential function.	**4.4.4a** An algebraic rule for an exponential function can be used to determine an exact output for a specified input and can be used to approximate an input for a specified output. **4.4.4b** A table of values for an exponential function can often be used to determine an exact or approximate output for a specified input or an exact or approximate input for a specified output. **4.4.4c** A graph of an exponential function can be used to approximate an output for a specified input and to approximate an input for a specified output.
4.4.5 Model a contextual scenario with an exponential function.	**4.4.5a** A contextual scenario that exhibits constant multiplicative growth or decay in its outputs over equal differences in the corresponding inputs can be modeled by an exponential function. **4.4.5b** Estimated inputs and outputs for exponential functions derived from a contextual scenario can be determined free of context, but the values must be interpreted in context to be correctly understood. **4.4.5c** The estimated inputs and outputs for an exponential function derived from a context should involve the same units as the variables in the contextual scenario.

Content Boundary: The focus in this key concept should be on having students use their knowledge of multiplicative patterns to explore simple exponential growth and decay relationships. Students will be expected to generate a table of values, construct a graph, and write an algebraic representation of an exponential function. Students should not solve problems involving formulas relating to geometric sequences, compound interest, or logarithms as these topics are beyond the scope of this course.

Pre-AP Algebra 1 Model Lessons

Model lessons in Pre-AP Algebra 1 are developed in collaboration with Algebra 1 educators across the country and are rooted in the course framework, shared principles, and areas of focus. Model lessons are carefully designed to illustrate on-grade-level instruction. Pre-AP strongly encourages teachers to internalize the lessons and then offer the supports, extensions, and adaptations necessary to help all students reach these on-grade-level goals.

The purpose of these model lessons is twofold:

- **Robust instructional support for teachers:** Pre-AP Algebra 1 model lessons are comprehensive lesson plans that, along with accompanying student resources, embody the Pre-AP approach to teaching and learning. Model lessons provide clear and substantial instructional guidance to support teachers as they engage students in the shared principles and areas of focus.
- **Key instructional strategies:** Commentary and analysis embedded in each lesson highlight not just what students and teachers do in the lesson, but also how and why they do it. This educative approach provides a way for teachers to gain unique insight into key instructional moves that are powerfully aligned with the Pre-AP approach to teaching and learning. In this way, each model lesson works to support teachers in the moment of use with students in their classroom.

Teachers have the option to use any or all model lessons alongside their own locally developed instructional resources. Model lessons target content areas that tend to be challenging for teachers and students. While the lessons are distributed throughout all four units, they are concentrated more heavily in the beginning of the course to support teachers and students in establishing a strong foundation in the Pre-AP approach to teaching and learning.

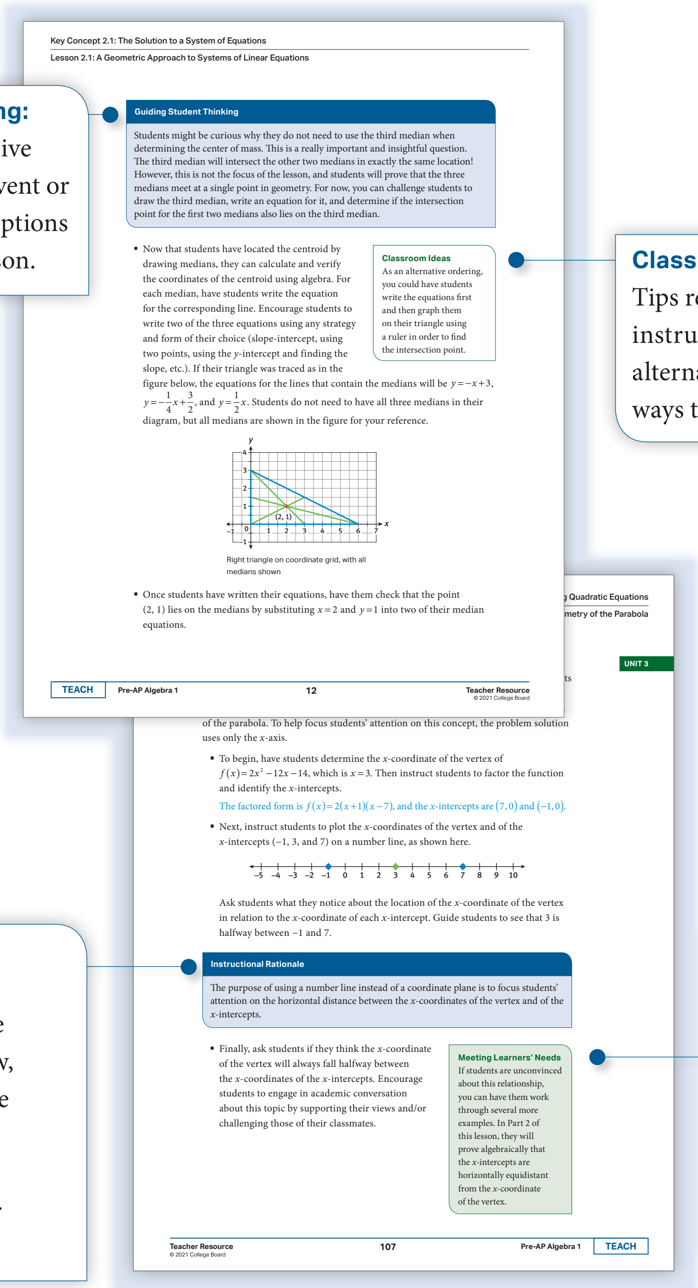
SUPPORT FEATURES IN MODEL LESSONS

The following support features recur throughout the Pre-AP Algebra 1 lessons, to promote teacher understanding of the lesson design and provide direct-to-teacher strategies for adapting lessons to meet their students' needs:

- Instructional Rationale
- Guiding Student Thinking
- Meeting Learners' Needs
- Classroom Ideas

Guiding Student Thinking: Ways to facilitate productive student thinking and prevent or address student misconceptions in critical areas of the lesson.

Classroom Ideas: Tips related to the logistics of the instruction, such as suggestions for alternative presentation methods or ways to alleviate pacing concerns.



Instructional Rationale: Insight into the strategic design and purpose of the instructional choices, flow, and scaffolding within the model lesson. Rationales often describe how a concept is continued later in the lesson or unit.

Meeting Learners' Needs: Optional differentiation strategies to address diverse learning needs, such as ideas for just-in-time skill building during a lesson or ways to break a task into smaller tasks, if needed, to make it more accessible.

Pre-AP Algebra 1 Assessments for Learning

Pre-AP Algebra 1 assessments function as a component of the teaching and learning cycle. Progress is not measured by performance on any single assessment. Rather, Pre-AP Algebra 1 offers a place to practice, to grow, and to recognize that learning takes time. The assessments are updated and refreshed periodically.

LEARNING CHECKPOINTS

Based on the Pre-AP Algebra 1 Course Framework, the learning checkpoints require students to examine data, models, diagrams, and short texts—set in authentic contexts—in order to respond to a targeted set of questions that measure students' application of the key concepts and skills from the unit. All eight learning checkpoints are automatically scored, with results provided through feedback reports that contain explanations of all questions and answers as well as individual and class views for educators. Teachers also have access to assessment summaries on Pre-AP Classroom, which provide more insight into the question sets and targeted learning objectives for each assessment event.

The following tables provide a synopsis of key elements of the Pre-AP Algebra 1 learning checkpoints.

Format	Two learning checkpoints per unit Digitally administered with automated scoring and reporting Questions target both concepts and skills from the course framework
Time Allocated	Designed for one 45-minute class period per assessment
Number of Questions	10–12 questions per assessment ▪ 7–9 four-option multiple choice ▪ 3–5 technology-enhanced questions

Domains Assessed	
Learning Objectives	Learning objectives within each key concept from the course framework
Skills	Three skill categories aligned to the Pre-AP mathematics areas of focus assessed regularly across all eight learning checkpoints: ▪ greater authenticity of applications and modeling ▪ engagement in mathematical argumentation ▪ connections among multiple representations

Question Styles	Question sets consist of two to three questions that focus on a single stimulus or group of related stimuli, such as diagrams, graphs, or tables. Questions embed mathematical concepts in real-world contexts. *Please see page 64 for a sample question set that illustrates the types of questions included in Pre-AP learning checkpoints and the Pre-AP final exam.*

PERFORMANCE TASKS

Each unit includes one performance-based assessment designed to evaluate the depth of student understanding of key concepts and skills that are not easily assessed in a multiple-choice format.

These tasks, developed for students across a broad range of readiness levels, are accessible while still providing sufficient challenge and the opportunity to practice the analytical skills that will be required in AP mathematics courses and for college and career readiness. Teachers participating in the official Pre-AP Program will receive access to online learning modules to support them in evaluating student work for each performance task.

Format	One performance task per unit Administered in print Educator scored using scoring guidelines
Time Allocated	Approximately 45 minutes or as indicated
Number of Questions	An open-response task with multiple parts

Domains Assessed	
Key Concepts	Key concepts and prioritized learning objectives from the course framework
Skills	Three skill categories aligned to the Pre-AP mathematics areas of focus: ▪ greater authenticity of applications and modeling ▪ engagement in mathematical argumentation ▪ connections among multiple representations

PRACTICE PERFORMANCE TASKS

One or more practice performance tasks in each unit provide students with the opportunity to practice applying skills and knowledge in a context similar to a performance task, but in a more scaffolded environment. These tasks include strategies for adapting instruction based on student performance and ideas for modifying or extending tasks based on students' needs.

SAMPLE PERFORMANCE TASK AND SCORING GUIDELINES

The following task and set of scoring guidelines are representative of what students and educators will encounter on the performance tasks. (The example below includes a portion of a practice performance task in Unit 1.)

PRACTICE PERFORMANCE TASK

Measuring the Wind Speed of a Hurricane

OVERVIEW

PERFORMANCE TASK DESCRIPTION

Part 1: Warming Up
Students get ready with a few questions about lines.

Part 2: Using Two Points to Write a Linear Formula
Students engage in a problem with two data points and write an equation from them.

Part 3: Additional Practice
Students have an opportunity to hone their skills.

Part 4: Summary
Students recap the important mathematics of the unit.

CONTENT FOCUS

This lesson is a synthesis of all prior lessons. Students calculate and interpret the rate of change of a relationship using two points, write a linear function that can be used to model the relationship, and use the function to answer questions about the quantities involved. The context for this lesson is the relationship between barometric pressure and wind speed during a hurricane.

AREAS OF FOCUS

- Greater Authenticity of Applications and Modeling
- Engagement in Mathematical Argumentation

SUGGESTED TIMING

~90 minutes

HANDOUTS

- Practice Performance Task: Measuring the Wind Speed of a Hurricane
- Practice Performance Task: Additional Practice Resources

Measuring the Wind Speed of a Hurricane

When measuring the wind speed of a hurricane or similar storm, meteorologists don't want to get hit by flying debris. They use a measure available to them in the comfort of their TV studio. Luckily for the meteorologists, the barometric pressure is linearly related to the wind speed.

Suppose scientists compared the wind speeds and barometric pressure readings of two severe storms over the Atlantic Ocean. For one of the storms, the barometric pressure was 1,000 mb (millibars), and the maximum wind speed was 100 kmh (kilometers per hour). The second had a barometric pressure of 960 mb and maximum wind speeds of 180 kmh.

(a) Develop a rule that you could use to predict the wind speed given any barometric pressure.

(b) Use your rule to predict the wind speed of a hurricane with a barometric pressure of 980 mb. Using what you know about linearity, explain why your prediction is reasonable.

(c) Interpret the intercept for wind speed in this context (i.e., as it pertains to the weather).

(d) Explain the meaning of slope in this context.

SCORING GUIDELINES

There are 12 possible points for this performance task.

Part (a)

Sample Solutions	Points Possible
Ordered pairs in the form (mb, kmh): (1,000, 100) and (960, 180). Let x be the barometric pressure and y be the wind speed. Slope $=\frac{(180-100)}{(960-1{,}000)}=-2$ Formula: $y = -2(x-1{,}000) + 100$	**3 points maximum** 1 point for correct ordered pairs in either (mb, kmh) or (kmh, mb) and for defining the variables *Scoring note:* Students should receive full credit if they correctly identify their choice for each variable. 1 point for correct slope calculation 1 point for correct formula (any equivalent form is acceptable)

Targeted Feedback for Student Responses
If students have difficulty with part (a), they may not be sure whether to use the barometric pressure or the wind speed for the independent variable. You could ask them some questions to get them to think about what they know and what they want to know. In this problem, it is probably more natural to choose barometric pressure as the independent variable because we want to use it to predict the wind speed. However, choosing the wind speed as the independent variable would be an acceptable choice for this problem.

TEACHER NOTES AND REFLECTIONS

Part (b)

Sample Solutions	Points Possible
$y = -2(980 - 1{,}000) + 100 = 140$ 140 kmh is reasonable. Because 980 mb is halfway between 960 mb and 1,000 mb, we should expect a wind speed halfway between 180 kmh and 100 kmh.	**3 points maximum** 1 point for correct calculation 2 points for clear and correct explanation

Targeted Feedback for Student Responses
If students have difficulty with part (b), they may not be sure how to explain why 140 kmh is a reasonable wind speed. You can use some questions to help them recognize that because the relationship between the barometric pressure and the wind speed is linear, we can use proportions to determine the reasonableness of the solution. That is, if the barometric pressure was closer to 1,000 mb than it was to 960 mb, then we would expect the wind speed to be closer to 100 kmh than to 180 kmh.

TEACHER NOTES AND REFLECTIONS

Part (c)

Sample Solutions	Points Possible
The wind speed intercept is (0, 2,100) [or (2,100, 0) if students ordered variables in that order]. This means that when the barometric pressure is 0 mb, then the wind speed would be 2,100 kmh. However, in the context of real weather, this is an impossible situation.	**3 points maximum** 1 point for the value of the wind speed intercept 1 point for explanation of intercept 1 point for explanation for weather context
Targeted Feedback for Student Responses	
If students have difficulty with part (c), they may not be used to analyzing the answer to a problem in math class in terms of its real-world context. You can help them by letting them know that all mathematical models make assumptions and that many models have limited contextual domains. The maximum recorded wind speed as of 2018 is just over 400 kmh, therefore 2,100 kmh is unreasonable.	

TEACHER NOTES AND REFLECTIONS

Part (d)

Sample Solutions	Points Possible
The slope means that every decrease of 2 kmh in wind speed corresponds to a 1 mb increase in barometric pressure.	**3 points maximum** 1 point for correct use of 2 kmh 1 point for correct use of 1 mb 1 point for correct correspondence of units
Targeted Feedback for Student Responses	
If students have difficulty with part (d), they would benefit from additional practice interpreting slope in context.	

TEACHER NOTES AND REFLECTIONS

Suggested point conversion, if assigning a grade to this problem:

Points Received	Appropriate Letter Grade (if Graded)	How Students Should Interpret Their Score
11 or 12 points	A	"I know all of this algebra really well."
8 to 10 points	B	"I know all of this algebra well, but I made a few mistakes."
5 to 7 points	C	"I know some of this algebra well, but not all of it."
2 to 4 points	D	"I only know a little bit of this algebra."
0 or 1 point	F	"I don't know much of this algebra at all."

FINAL EXAM

Pre-AP Algebra 1 includes a final exam featuring multiple-choice and technology-enhanced questions as well as an open-response question. The final exam is a summative assessment designed to measure students' success in learning and applying the knowledge and skills articulated in the Pre-AP Algebra 1 Course Framework. The final exam's development follows best practices such as multiple levels of review by educators and experts in the field for content accuracy, fairness, and sensitivity. The questions on the final exam have been pretested, and the resulting data are collected and analyzed to ensure that the final exam is fair and represents an appropriate range of the knowledge and skills of the course.

The final exam is designed to be delivered on a secure digital platform in a classroom setting. Educators have the option of administering the final exam in a single extended session or two shorter consecutive sessions to accommodate a range of final exam schedules.

Multiple-choice and technology-enhanced questions are delivered digitally and scored automatically with detailed score reports available to educators. This portion of the final exam is designed to build on the question styles and formats of the learning checkpoints; thus, in addition to their formative purpose, the learning checkpoints provide practice and familiarity with the final exam. The open-response question, modeled after the performance tasks, is delivered as part of the digital final exam but is designed to be scored separately by educators using scoring guidelines that are designed and vetted with the question.

The tables on the following page provide a synopsis of key elements of the Pre-AP Algebra 1 Final Exam.

Format	Digitally administered Questions target both concepts and skills from the course framework Specific questions are graphing calculator enabled
Time Allocated	One 105-minute session or two sessions of 60 minutes and 45 minutes
Number of Questions	35–40 questions ▪ four-option multiple-choice questions ▪ technology-enhanced questions ▪ one multipart open-response question
Scoring	▪ automatic scoring for multiple-choice and technology-enhanced questions ▪ educator scoring for open-response questions ▪ comprehensive score reports with individual student and class views for educators

Domains Assessed	
Content	Key concepts and prioritized learning objectives from the course framework
Skills	Three skill categories aligned to the Pre-AP mathematics areas of focus assessed in the final exam: ▪ greater authenticity of applications and modeling ▪ engagement in mathematical argumentation ▪ connections among multiple representations

Question Styles	Question sets consist of two to three questions that focus on a single stimulus or group of related stimuli, such as diagrams, graphs, or tables. Questions embed mathematical concepts in real-world contexts. *Please see page 64 for a sample question set that illustrates the types of questions included in Pre-AP learning checkpoints and the Pre-AP final exam.*

SAMPLE ASSESSMENT QUESTIONS

The following questions are representative of what students and educators will encounter on the learning checkpoints and final exam.

1. Jazmin runs a lemonade stand every Saturday. She sells each cup of lemonade for the same price. Which of the following statements describes how Jazmin can most accurately model her daily revenue?

   (A) She can use a linear function to model her revenue because the rate of change, in terms of dollars earned per cup of lemonade sold, is constant.

   (B) She can use a quadratic function to model her revenue because the rate of change, in terms of dollars earned per cup of lemonade sold, is linear.

   (C) She can use an exponential function to model her revenue because the growth factor, in terms of dollars earned per cup of lemonade sold, is constant.

   (D) She can use a piecewise linear function to model her revenue because the rate of change, in terms of dollars earned per cup of lemonade sold, varies over the course of the day.

Assessment Focus

Question 1 assesses whether a student can identify an appropriate function to use for modeling a rate of change when given a real-world scenario.

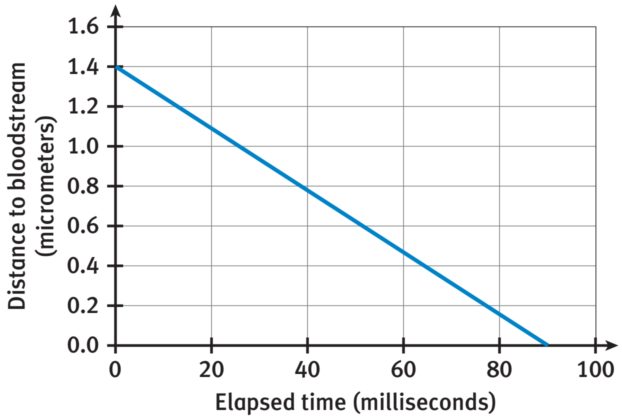
Correct Answer: A

Learning Objective:

1.1.4 Determine whether a relationship presented graphically or numerically is linear by examining the rate of change.

Area of Focus: Engagement in Mathematical Argumentation

2. In humans, oxygen molecules move from the lungs into the bloodstream, where they are carried throughout the body. The graph below shows the distance, in micrometers, an oxygen molecule is from the bloodstream and the elapsed time, in milliseconds, it took to move from the lungs into the bloodstream.



What is the value of the vertical axis intercept and what does it mean?

(A) 1.4 milliseconds is the time it took the oxygen molecule to move from the lungs into the bloodstream.

(B) 90 milliseconds is the time it took the oxygen molecule to move from the lungs into the bloodstream.

(C) 90 micrometers is the distance the oxygen molecule moved between the lungs and the bloodstream.

(D) 1.4 micrometers is the distance the oxygen molecule moved between the lungs and the bloodstream.

Assessment Focus

Question 2 assesses whether or not students can interpret the vertical axis intercept in context. They must apply the skill of interpreting information presented in a graph in order to solve the problem.

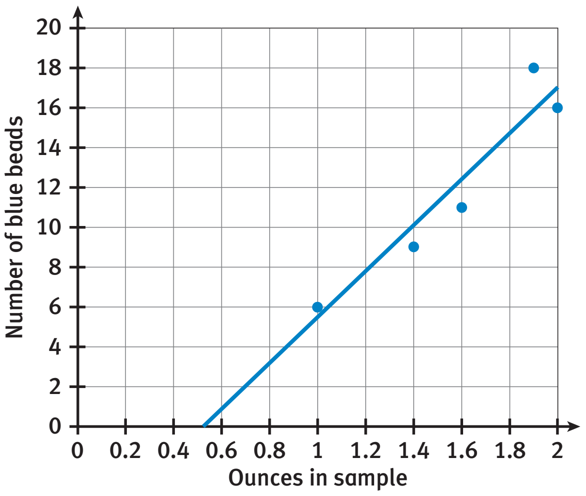
Correct Answer: D

Learning Objective:

1.2.6 Model a contextual scenario with a linear function.

Area of Focus: Greater Authenticity of Applications and Modeling

3. Eli is purchasing beads for an art project. The supply store sells the beads in bulk in multicolor assortments. Eli only wants to use blue beads for his project, but the store won't allow him to pick through the beads to select only blue ones. From the large bin of assorted beads, he takes five small samples of varying weight and counts the number of blue beads in each sample. The scatterplot represents his findings.



To determine an equation for a trend line, Eli uses two points that he thinks lie on the line: the number of blue beads for a sample that measures 0.6 ounces and the number of blue beads for a sample that measures 2 ounces. Which of the following is the correct equation of the trend line that Eli determined?

(A) $y=\left(\frac{16}{1.4}\right)x-\frac{41}{7}$

(B) $y=\left(\frac{15}{1.4}\right)x-\frac{38}{7}$

(C) $y=\left(\frac{1.4}{16}\right)x+\frac{673}{400}$

(D) $y=\left(\frac{1.4}{15}\right)x-\frac{1186}{75}$

Assessment Focus

Question 3 assesses whether or not students can determine the equation of the line of best fit. Students must also translate between information provided in the stimulus text and the scatterplot.

Correct Answer: A

Learning Objective:

1.4.4 Determine an equation for a trend line that describes trends in a scatterplot.

Area of Focus: Connections Among Multiple Representations

4. Laura is a general contractor who needs to order at least 110 sheets of plywood for a house she is building. There are two supply companies, Menureds and HouseCo, and neither one has enough plywood in stock for Laura's project. Menureds sells plywood in bundles of 3 sheets and they have 12 bundles available at $72 per bundle. HouseCo sells plywood in bundles of 4 sheets and they have 25 bundles available at $76 per bundle.

   Laura has budgeted up to $2,200 for plywood. Which combination of bundles gets Laura at least 110 sheets and and stays under budget?

   (A) 12 bundles from Menureds and 18 bundles from HouseCo

   (B) 10 bundles from Menureds and 20 bundles from HouseCo

   (C) 12 bundles from Menureds and 25 bundles from HouseCo

   (D) 5 bundles from Menureds and 24 bundles from HouseCo

Assessment Focus

Question 4 assesses whether or not students can represent and model a scenario involving a simple system of inequalities and interpret potential solutions.

Correct Answer: D

Learning Objective:

2.4.3 Model a contextual scenario with a system of linear inequalities.

Area of Focus: Greater Authenticity of Applications and Modeling

Pre-AP Algebra 1 Course Designation

Schools can earn an official Pre-AP Algebra 1 course designation by meeting the program commitments summarized below. Pre-AP Course Audit Administrators and teachers will complete a Pre-AP Course Audit process to attest to these commitments. All schools offering courses that have received a Pre-AP Course Designation will be listed in the Pre-AP Course Ledger, in a process similar to that used for listing authorized AP courses.

PROGRAM COMMITMENTS

- Teachers have read the most recent *Pre-AP Algebra 1 Course Guide*.
- The school ensures that Pre-AP frameworks and assessments serve as the foundation for all sections of the course at the school. This means that the school must not establish any barriers (e.g., test scores, grades in prior coursework, teacher or counselor recommendation) to student access and participation in the Pre-AP Algebra 1 coursework.
- Teachers administer at least one of two learning checkpoints per unit on Pre-AP Classroom and one performance task per unit.
- Teachers complete the foundational professional learning (Online Foundational Modules or Pre-AP Summer Institute) and at least one online performance task scoring module. The current Pre-AP coordinator completes the Pre-AP Coordinator Online Module.
- Teachers align instruction to the Pre-AP Algebra 1 Course Framework and ensure their course meets the curricular commitments summarized below.
- The school ensures that the resource commitments summarized below are met.

CURRICULAR COMMITMENTS

- The course provides opportunities for students to develop understanding of the Pre-AP Algebra 1 key concepts and skills articulated in the course framework through the four units of study.
- The course provides opportunities for students to engage in the Pre-AP shared instructional principles.
    - close observation and analysis
    - evidence-based writing
    - higher-order questioning
    - academic conversation

- The course provides opportunities for students to engage in the three Pre-AP Algebra 1 areas of focus. The areas of focus are:
  - greater authenticity of applications and modeling
  - engagement in mathematical argumentation
  - connections among multiple representations
- The instructional plan for the course includes opportunities for students to continue to practice and develop disciplinary skills.
- The instructional plan reflects time and instructional methods for engaging students in reflection and feedback based on their progress.
- The instructional plan reflects making responsive adjustments to instruction based on student performance.

RESOURCE REQUIREMENTS

- The school ensures that participating teachers and students are provided computer and internet access.
- Teachers should have consistent access to a video projector for sharing web-based instructional content and short web videos.

Accessing the Digital Materials

Pre-AP Classroom is the online application through which teachers and students can access Pre-AP instructional resources and assessments. The digital platform is similar to AP Classroom, the online system used for AP courses.

Pre-AP coordinators receive access to Pre-AP Classroom via an access code delivered after orders are processed. Teachers receive access after the Pre-AP Course Audit process has been completed.

Once teachers have created course sections, student can enroll in them via access code. When both teachers and students have access, teachers can share instructional resources with students, assign and score assessments, and complete online learning modules; students can view resources shared by the teacher, take assessments, and receive feedback reports to understand progress and growth.

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